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FUZZY ROUGH CONNECTED SPACES

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ABSTRACT : In this paper we introduce the concept of connectedness in fuzzy rough topological spaces. We also investigate some properties of connectedness in fuzzy rough topological spaces.

KEYWORDS - Fuzzy rough open and closed sets, fuzzy rough subspace, fuzzy rough closure, fuzzy rough connected space.

I. INTRODUCTION

The theory of rough sets was proposed by Pawlak [1,2]. It is an extension of set theory for the study of systems characterized by insufficient and incomplete informations. A key notion in Pawlak rough set model is equivalence relation. The equivalence classes form the building blocks for the construction of lower and upper approximations. Replacing the equivalence relation by an arbitrary binary relation, different kinds of generalizations in Pawlak rough set models were obtained. In [3,4,5], the concept of fuzzy rough sets were studied by replacing crisp binary relations with fuzzy relations on the universe. Yong Chan Kim [6] introduced the separation axioms of fuzzy topological spaces using R- fuzzy semi-open (closed) sets.

Keyun Qin and Pei [7] discussed the relationship between fuzzy rough set models and fuzzy topologies on a finite universe. Mathew and John [8] developed general topological structures on rough sets. Lellis Thivagar et al. [9] introduced a new topology called rough topology In terms of rough sets and showed that rough topology can be used to analyze many real life problems. Samanta et al. [10] defined fuzzy rough relation on a set and proved that the collection of such relations is closed under different binary compositions such as algebraic sum, algebraic product, etc. Fatima et al. [11] introduced totally continuous functions and totally semi continuous functions in double fuzzy topological spaces and investigated some of their characterizations.

In this paper we introduce the concept of connectedness in fuzzy rough topological spaces. We also investigate some properties of connectedness in fuzzy rough topological spaces.

II. PRELIMINARIES

In this section we recall some useful definitions and results.

Notation. Throughout this paper U denotes a non-empty finite universe and $F(U)$ denotes the set of all fuzzy subsets of U.

Definition 2.1. [12] A fuzzy subset A of the universe U is characterized by a membership function $\mu_A : U \to [0,1]$.

Definition 2.2. [12] A fuzzy relation R on A is said to be reflexive if and only if $\forall\, x \in U$, $\mu_A(x) > 0$ and $\mu_R(x,x) = 1$.

Definition 2.3 [12] A fuzzy relation R on A is said to be symmetric if and only if $\mu_R(x,y) = \mu_R(y,x) \forall x, y \in U$.

Definition 2.4 [12] A fuzzy relation R on A is said to be transitive if and only if $\mu_R(x,z) \geq \max_{y\in U} \min\{\mu_R(x,y), \mu_R(y,z)\}$.

Definition 2.5 [12] A fuzzy relation that is reflexive, symmetric and transitive is called a fuzzy equivalence relation.

Definition 2.6 [3] Let U be a nonempty and finite universe of discourse , and FR be a fuzzy equivalence relation defined on $U \times U$. The pair (U, FR) is called a fuzzy approximation space. For any $A \in F(U)$, the upper and lower approximations of A with $FR(A)$ respect to (U, FR), denoted by $F\underline{R}(A)$ and $F\bar{R}(A)$ are two fuzzy sets defined as:

$$F\underline{R}(A) = \{(x, \mu_{F\underline{R}(A)}(x)) \mid x \in U\},$$
$$F\bar{R}(A) = \{(x, \mu_{F\bar{R}(A)}(x)) \mid x \in U\}.$$

where

$$\mu_{F\underline{R}(A)}(x) = \wedge_{u\in U}((1-\mu_R(x,u)) \vee \mu_A(U)), x \in U,$$
$$\mu_{F\bar{R}(A)}(x) = \vee_{u\in U}(\mu_R(x,u) \wedge \mu_A(U)), x \in U.$$

The pair $FR(A) = (F\underline{R}(A), F\bar{R}(A))$ is called the fuzzy rough set of A with respect to (U, FR)

Definition 2.7 Let $FR(A)$ and $FR(B)$ be two fuzzy rough sets. $FR(A)$ is said to be a fuzzy rough subset of $FR(B)$ denoted by $FR(A) \subseteq FR(B)$ if $\mu_{F\underline{R}(A)}(x) \geq \mu_{F\underline{R}(B)}(x)$ and $\mu_{F\bar{R}(A)}(x) \leq \mu_{F\bar{R}(B)}(x), x \in U$.

Definition 2.8. [7] Let U be the universe , $X \subseteq U$. R be an equivalence relation on U and $\tau_R = \{U, \phi, \underline{R}(X), \bar{R}(X)\}$. τ_R satisfies the following axioms:

1. U and $\phi \in \tau_R$.
2. Union of the elements of any sub collection of τ_R is in τ_R.
3. Intersection of the elements of any finite sub collection of τ_R is in τ_R.

τ_R forms a topology on U called as a rough topology on U with respect to X. We call (U, X, τ_R) as a rough topological space.

Definition 2.9. [10] For $\lambda, \mu \in I^X, \lambda$ is called quasi coincident with μ, denoted by $\lambda q \mu$ if there exists $x \in X$ such that $\lambda(x) + \mu(x) > 1$. Otherwise it is denoted by $\lambda \bar{q} \mu$.

Definition 2.10. [11] For $x \in X$ and $t \in I_0 = (0,1]$, a fuzzy point is defined by

$$x_t(y) = \begin{cases} t & \text{if } y = x, \\ 0 & \text{if } y \neq x. \end{cases}$$

$x_t \in \lambda \in I^X$ if and only if $t \leq \lambda(x)$.

III. FUZZY ROUGH CONNECTED SPACES

In this section we define fuzzy rough connected spaces and also investigate some properties of these spaces.

Definition 3.1. Let U be the universe and $A \subseteq U$. FR be a fuzzy equivalence relation on U and $\tau = \{U, \phi, \mathrm{F}\underline{R}(A), \mathrm{F}\overline{R}(A)\}$. τ satisfies the following axioms:

1. U and $\phi \in \tau$.
2. The union of the elements of any sub collection of τ is in τ.
3. The intersection of the elements of any finite sub collection of τ is in τ.

   τ forms a topology called the rough topology on U with respect to A and $(U, \mathrm{FR}(A), \tau)$ is called as the fuzzy rough topological space.

Definition 3.2. If τ is a fuzzy rough topology on U, then any member of τ is called fuzzy rough set in U.

Definition 3.3. The complement of a fuzzy rough open set A is called a fuzzy rough closed set and is denoted by $[FR(A)]^c$.

Definition 3.4. The fuzzy rough set $FR(A)$ over U is called a fuzzy rough point in U denoted by $xFR(A)$, if for the element $x \in U, \mu_{FR(A)}(x) \neq o, \mu_{FR(A)}(x') = 0 \ \forall \ x' \in U - \{x\}$.

Definition 3.5. A fuzzy rough set $FR(E)$ in a fuzzy rough topological space $(U, FR(A), \tau)$ is called a fuzzy rough neighborhood of the fuzzy rough point $xFR(P)$ for some $x \in U$, if there is a fuzzy rough open set $FR(D)$ such that $xFR(P) \in FR(D) \subseteq FR(E)$.

Definition 3.6. Let $(U, FR(A), \tau)$ be a fuzzy rough topological space and $FR(A)$ be a fuzzy rough set over U. The fuzzy rough closure of $FR(A)$ is defined as the intersection of all fuzzy rough closed sets which contain $FR(A)$ and is denoted by $\overline{FR(A)}$. i.e. $\overline{FR(A)} = \cap\{FR(B) : FR(B)$ is fuzzy rough closed set and $FR(A) \subseteq FR(B)\}$.

Definition 3.7. A fuzzy rough point $xFR(P)$ is said to belong to a fuzzy rough set $FR(B)$ denoted by $xFR(P) \in FR(B)$ if $xFR(P) \leq \mu_{FR(B)}(x), \ \forall \ x \in U$.

Definition 3.8. Two fuzzy rough sets $FR(A)$ and $FR(B)$ are said to be quasi coincident denoted by $FR(A)qFR(B)$ if there exists $x \in U \ni \mu_{FR(A)}(x) + \mu_{FR(B)}(x) > 1$. Otherwise $FR(A)\bar{q}FR(B)$, in which case $\mu_{FR(A)}(x) + \mu_{FR(B)}(x) \leq 1, \ \forall \ x \in U$.

Definition 3.9. Let $\left(U, FR(A), \tau\right)$ be a fuzzy rough topological space and $FR(B_S) \subset FR(A)$. Then the fuzzy rough topology $\tau_{FR(B_S)} = \left\{FR(B_S) \cap FR(C) \mid FR(C) \in \tau\right\}$ is called fuzzy rough subspace topology and $\left(FR(B_S), \tau_{FR(B_S)}\right)$ is called fuzzy rough subspace of $\left(U, FR(A), \tau\right)$.

Definition 3.10. Let τ_1 and τ_2 be two fuzzy rough topologies on (U, FR). τ_1 is said to be coarser than τ_2, if $\tau_1 \subset \tau_2$ and in this case τ_2 is said to be finer than τ_1. If either $\tau_1 \subset \tau_2$ or $\tau_2 \subset \tau_1$, then the topologies τ_1 and τ_2 are said to be comparable. If $\tau_1 \not\subset \tau_2$ and $\tau_2 \not\subset \tau_1$, then the topologies τ_1 and τ_2 are not comparable.

Definition 3.11. A fuzzy rough topological space is said to be separated if there exists a fuzzy rough open sets $FR(A), FR(B)$ such that $FR(A)\bar{q}FR(B)$, i.e. $\mu_{FR(A)}(x) + \mu_{FR(B)}(x) \le 1 \ \forall \ x \in U$.

If there does not exist a fuzzy rough separation of U, then the fuzzy rough topological space is said to be fuzzy rough connected.

Definition 3.12. Consider $U = \{x_1, x_2, x_3, x_4, x_5\}$,
$A = \{(x_1, 0.8), (x_2, 0.9), (x_3, 0.95), (x_4, 0.96), (x_5, 0.98)\}$ and

$$FR = \begin{pmatrix} 1 & 0.92 & 0.96 & 0.98 & 1 \\ 0.92 & 1 & 0.85 & 0.8 & 0.75 \\ 0.96 & 0.85 & 1 & 0.76 & 0.6 \\ 0.98 & 0.8 & 0.76 & 1 & 0.5 \\ 1 & 0.75 & 0.6 & 0.5 & 1 \end{pmatrix}$$

Now,

$\mu_{F\underline{R}(A)}(x) = \bigwedge_{u \in U} [(1 - \mu_{FR}(x,u)) \vee \mu_A(u)]$ and

$\mu_{F\bar{R}(A)}(x) = \bigvee_{u \in U} [\mu_{FR}(x,u) \wedge \mu_A(u)]$

The lower approximations are

$\mu_{F\underline{R}(A)}(x_1) = 0.8,\ \mu_{F\underline{R}(A)}(x_2) = 0.8,\ \mu_{F\underline{R}(A)}(x_3) = 0.8,\ \mu_{F\underline{R}(A)}(x_4) = 0.8,\ \mu_{F\underline{R}(A)}(x_5) = 0.8.$

$F\underline{R}(A) = \{x_1/0.8,\ x_2/0.8,\ x_3/0.8,\ x_4/0.8,\ x_5/0.8\}.$

The upper approximations are

$\mu_{F\bar{R}(A)}(x_1) = 0.98,\ \mu_{F\bar{R}(A)}(x_2) = 0.9,\ \mu_{F\bar{R}(A)}(x_3) = 0.95,$

$\mu_{F\bar{R}(A)}(x_4) = 0.96,\ \mu_{F\bar{R}(A)}(x_5) = 0.98.$

$F\underline{R}(A) = \{x_1/0.98,\ x_2/0.9,\ x_3/0.95,\ x_4/0.96,\ x_5/0.98\}.$

Consider,

$U = \{x_1, x_2, x_3, x_4, x_5\},\ \ B = \{(x_1, 0.4), (x_2, 0.3), (x_3, 0.6), (x_4, 0.5), (x_5, 0.45)\}$

and $FR = \begin{pmatrix} 1 & 0.4 & 0.6 & 0.7 & 0.75 \\ 0.4 & 1 & 0.5 & 0.35 & 0.75 \\ 0.6 & 0.5 & 1 & 0.8 & 0.65 \\ 0.7 & 0.35 & 0.8 & 1 & 0.55 \\ 0.75 & 0.75 & 0.65 & 0.55 & 1 \end{pmatrix}$

The lower approximations are

$\mu_{F\underline{R}(B)}(x_1) = 0.4,\ \mu_{F\underline{R}(B)}(x_2) = 0.3,\ \mu_{F\underline{R}(B)}(x_3) = 0.4,\ \mu_{F\underline{R}(B)}(x_4) = 0.4,\ \mu_{F\underline{R}(B)}(x_5) = 0.3.$

$F\underline{R}(B) = \{x_1/0.4, x_2/0.3, x_3/0.4, x_4/0.4, x_5/0.3\}.$

The upper approximations are

$\mu_{F\bar{R}(B)}(x_1) = 0.6,\ \mu_{F\bar{R}(B)}(x_2) = 0.45,\ \mu_{F\bar{R}(B)}(x_3) = 0.6,\ \mu_{F\bar{R}(B)}(x_4) = 0.6,\ \mu_{F\bar{R}(B)}(x_5) = 0.6.$

$F\bar{R}(B) = \{x_1/0.6, x_2/0.45, x_3/0.6, x_4/0.6, x_5/0.6\}.$

$\mu_{F\underline{R}(A)}(x_1) + \mu_{F\underline{R}(B)}(x_1) = 1.2 > 1$

$\mu_{F\bar{R}(A)}(x_1) + \mu_{F\bar{R}(B)}(x_1) = 1.58 > 1.$

Similarly, other values can be found.

Clearly $(U, FR(A), \tau)$ is fuzzy rough connected.

Theorem:3.13. A fuzzy rough topological space $(U, FR(A), \tau)$ is fuzzy rough separated if and only if there exists a non empty proper fuzzy rough subset of $FR(A)$ which is both fuzzy rough open and fuzzy rough closed.

Proof. Let $FR(B_s)$ be a non empty proper fuzzy rough subset of $FR(A)$ which is both fuzzy rough open and fuzzy rough closed. Now $FR(H_s) = [FR(B_s)]^c$ is a non empty proper fuzzy rough subset of $FR(A)$ which is also both fuzzy rough open and fuzzy rough closed.

$\Rightarrow \overline{FR(B_s)} = FR(B_s)$ and $\overline{FR(H_s)} = FR(H_s)$

$\Rightarrow FR(A)$ can be expressed as the union of two fuzzy rough sets $FR(B_s), FR(H_s)$

Such that $FR(B_s)\bar{q}FR(H_s)$

$\Rightarrow (U, FR(A), \tau)$ is fuzzy rough separated.

Conversely, let $(U, FR(A), \tau)$ be fuzzy rough separated.

$\Rightarrow \exists$ non empty fuzzy rough subsets $FR(B_s)$ and $FR(H_s) \ni \overline{FR(B_s)}\bar{q}FR(H_s)$ and $FR(B_s)\bar{q}\,\overline{FR(H_s)}$. Now, $FR(B_s) \subseteq \overline{FR(B_s)}$ and $\overline{FR(B_s)}\bar{q}FR(H_s)$.

$\Rightarrow FR(B_s)\bar{q}FR(H_s)$

$\Rightarrow FR(H_s)=[FR(B_s)]^c$. Also, $FR(B_s)\bar{q}\overline{FR(H_s)}$

$\Rightarrow FR(B_s)=[\overline{FR(H_s)}]^c$.

Similarly $FR(H_s)=[\overline{FR(B_s)}]^c$.

Hence $FR(B_s), FR(H_s)$ are fuzzy rough open sets being the complements of fuzzy rough closed sets. Also $FR(H_s)=[FR(B_s)]^c$

$\Rightarrow FR(H_s)$ is fuzzy rough closed.

Theorem 3.14. If the fuzzy rough sets $FR(A)$ and $FR(B)$ form a fuzzy rough separation of U and if $(FR(B_s), \tau_{FR(B_s)})$ is a fuzzy rough connected subspace of $(U, FR(A), \tau)$ then $FR(B_s)\subset FR(A)$ or $FR(B_s)\subset FR(B)$.

Proof. Suppose $FR(B_s)$ is neither contained in $FR(A)$ nor in $FR(B)$. Since $FR(A)$ and $FR(B)$ form a fuzzy rough separation of U, $FR(A)\bar{q}FR(B_s)$ and $FR(B)\bar{q}FR(B_s)$.

$(FR(B_s), \tau_{FR(B_s)})$ is a fuzzy rough subspace of $(U, FR(A), \tau)$. $FR(A)\cap FR(B_s)$ and $FR(B)\cap FR(B_s)$ are both open in $\tau_F R(B_s)$. $FR(A)$ and $FR(B)$ are fuzzy rough open sets $\ni FR(A)\bar{q}FR(B)$.

Consider, $(FR(A)\cap FR(B_s))\cap(FR(B)\cap FR(B_s))$

$=(FR(A)\cap FR(B))\cap FR(B_s)\subseteq FR(A)\cap FR(B)$,

where $FR(A)\bar{q}FR(B)$ $\Rightarrow (FR(A)\cap FR(B_s))\bar{q}(FR(B)\cap FR(B_s))$.

Hence $(FR(B_s), \tau_F R(B_s))$ is separated, a contradiction. Therefore $FR(B_s)\subset FR(A)$ or $FR(B_s)\subset FR(B)$.

Theorem.3.15. Let $(FR(B_s), \tau_{FR(B_s)})$ be a fuzzy rough connected subspace of $(U, FR(A), \tau)$. If $FR(B_s)\subset FR(A)\subset \overline{FR(B_s)}$, then $FR(A)$ is also fuzzy rough connected.

Proof. Let $FR(A)$ be connected and $FR(B_s)\subset FR(A)\subset \overline{FR(B_s)}$. Suppose not, then $FR(A)$ is separated. Therefore $\exists$ fuzzy rough open sets $FR(B)$ and $FR(C)\ni FR(B)\bar{q}FR(C)$. Then $FR(B_s)\subset FR(B)$ or $FR(B_s)\subset FR(C)$. If, $FR(B_s)\subset FR(B)$, then

$\overline{FR(B_s)}\subset \overline{FR(B)}$. Since $FR(B)\bar{q}FR(C), FR(B_s)$ cannot intersect $FR(C)$. This contradicts the fact that $FR(C)$ is nonempty. Hence $FR(A)$ is connected.

Remark. In particular if $FR(B_s)$ is fuzzy rough connected then $\overline{FR(B_s)}$ is fuzzy rough connected.

Theorem 3.16. Arbitrary union of fuzzy rough connected subspaces of $(U, FR(A), \tau)$ that have nonempty intersection is fuzzy rough connected.

Proof. Let $\{FR(B_{s_\lambda}), \tau_{FR(B_{s_\lambda})} \mid \lambda \in \Delta\}$ be a collection of fuzzy rough connected subspaces of $(U, FR(A), \tau)$ with nonempty intersection. To prove that $\cup FR(B_{s_\lambda})$ is connected. Suppose not, then $\cup FR(B_{s_\lambda})$ is separated. Therefore $\exists$ non empty disjoint fuzzy rough open sets $FR(A)$ and $FR(B) \ni FR(A)\overline{q}FR(B_{s_\lambda})$ and $FR(B)\overline{q}FR(B_{s_\lambda}),$ for each λ in the subspace. As $FR(B_{s_\lambda})$ is connected for each $\lambda,$ one of $FR(A)\overline{q}FR(B_{s_\lambda})$ and $FR(B)\overline{q}FR(B_{s_\lambda}).$

Suppose $FR(A)\overline{q}FR(B_{s_\lambda})$

$$\Rightarrow FR(B) \cap FR(B_{s_\lambda}) = FR(B_{s_\lambda})$$

$$\Rightarrow FR(B_{s_\lambda}) \subset FR(B)\ \forall\ \lambda \in \Delta$$

$$\Rightarrow \bigcup_{\lambda\in\Delta} FR(B_{s_\lambda}) \subset FR(B).$$

$$\Rightarrow FR(A) \cup FR(B) \subset FR(B).$$

$\Rightarrow FR(A)$ is empty, a contradiction.

Theorem.3.17. Arbitrary union of family of fuzzy rough connected subsets of $(U, FR(A), \tau)$ such that one of the members of the family has nonempty intersection with every member of the family, is fuzzy rough connected.

Proof. Let $\{FR(B_{s_\lambda}), \tau_{FR(B_{s_\lambda})} \mid \lambda \in \Delta\}$ be a collection of fuzzy rough connected subspaces of $(U, FR(A), \tau)$ and $FR(B_{S_{\lambda_0}})$ be a fixed member such that $FR(B_{S_{\lambda_0}})\overline{q}FR(B_{s_\lambda})$ for each $\lambda \in \Delta.$ Then by Theorem 3.16,

$FR(A_\lambda) = FR(B_{S_{\lambda_0}}) \cup FR(B_{s_\lambda})$ is fuzzy rough connected for each $\lambda \in \Delta.$ Now,

$$\bigcup_{\lambda\in\Delta} FR(A_\lambda) = \bigcup_{\lambda\in\Delta}(FR(B_{S_{\lambda_0}})\bigcup_{\lambda\in\Delta} FR(B_{s_\lambda})) = \bigcup_{\lambda\in\Delta} FR(B_{s_\lambda}) \text{ and}$$

$$\bigcap_{\lambda\in\Delta} FR(A_\lambda) = \bigcap_{\lambda\in\Delta}(FR(B_{S_{\lambda_0}})\bigcup_{\lambda\in\Delta} FR(B_{s_\lambda})) = FR(B_{s_{\lambda_0}})\bigcap_{\lambda\in\Delta}(\bigcup FR(B_{s_{\lambda_0}})) \neq \phi.$$

$\Rightarrow FR(A_\lambda)$ is fuzzy rough connected.

Theorem. 3.18. If $(U, FR(A), \tau_1)$ is a fuzzy rough connected space and τ_2 is fuzzy rough coarser than $\tau_1,$ then $(U, FR(A), \tau_2)$ is also fuzzy rough connected.

Proof. Assume that $FR(B), FR(C)$ form a fuzzy rough separation of $(U, FR(A), \tau_2)$.

$FR(B), FR(C) \in \tau_2, \Rightarrow FR(B), FR(C) \in \tau_1$

$\Rightarrow FR(B), FR(C)$ form a fuzzy rough separation of $(U, FR(A), \tau_1)$, a contradiction.

Therefore $(U, FR(A), \tau_2)$ is fuzzy rough connected.

IV. CONCLUSION

The notion of fuzzy rough connectedness captures the idea of “hanging-togetherness” of image elements in an object by assigning a strength of connectedness to every possible path between every possible pair of image elements. Fuzzy rough connectedness is a very useful topological property for a space to possess. Measures of fuzzy rough connectedness can be applied in image segmentation and network literature. The concept of fuzzy rough connectedness can be extended to fuzzy soft rough connectedness.

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REFERENCES

[1] Z.Pawlak, Rough sets, *International Journal of computer and Information sciences, 11* (1982), 341-356.

[2] Z.Pawlak, Rough Sets: Theoretical Aspects of Reasoning About Data, *Kluwer Academic Publishers, Boston,* 1991.

[3] D.Dubois, H.Prade, Rough fuzzy set and fuzzy rough sets, *Internat. J. Gen. Systems, 17* (1990), 191-209.

[4] D.Dubois, H.Prade, Putting fuzzy sets and rough sets together, *Intelligent Decision Support, Kluwer Academic, Dordrecht,* 1992, 203-232.

[5] S.Nanda, Fuzzy rough sets, *Fuzzy Sets and Systems, 45* (1992), 157-160.

[6] Yong Chan Kim and S.E.Abbas, Separations based on R-fuzzy Semi-open sets, *Int. J of Fuzzy Mathematics,15* (2007), 53-68.

[7] Keyun Qin and Zheng Pei, On the topological properties of fuzzy rough sets, *Fuzzy Sets and Systems, 151* (2005), 601-613.

[8] Boby P.Mathew and Sunil Jacob John, On rough topological spaces, *International Journal of Mathematical Archive, 3 (9)* (2012), 3413-3421.

[9] M.Lellis Thivagar, Carmel Richard and Nirmala Rebecca Paul, Mathematical Innovations of a Modern Topology in Medical Events, *International Journal of Information science, 2(4)* (2012), 33-36.

[10] T.K.Samanta and Biswajit Sarkar, *Fuzzy rough relations, arxiv:1109.1803, 1* (2011), 10-17.

[11] Fatimah M Mohammed, M.S.M Noorani, A.Ghareeb, it Several types of totally continuous functions in double fuzzy topological spaces, *Advances in fuzzy systems*, 2014, 361-398.

[12] George J.Klir, Tina A.Folger, Fuzzy sets, Uncertainty and information, Prentice Hall of India, (2005).