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Canonical pair of electric flux and magnetic flux in Bohr atom

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Abstract : *The quantum electromagnetic flux is studied as operators for magnetic flux quantization in the Bohr atom. We find that this quantization rule can be found from an elementary analysis of a Bohr electric oscillator treated like an L-C circuit. The electromagnetic flux quantization agrees for instanton configuration so there is no radiation in the Bohr atom and we have a stable atomic system .*

Keywords *:Quantum optics, L-C circuit quantization, Bohr model*

I. Introduction

The magnetic flux quantization (M.F.Q) was studied by London about the properties of the wave function phase [1, 2], leading to the experimental discovery of the M.F.Q [3]. Here, the wave function phase is given by an integration of the vector potential along a path with arbitrary initial point, and end point exactly where we have the argument of the wave function [4]. In a ring this approach gives the magnetic flux quantization

$$\oint A.dr = \phi = \frac{2\pi}{e}n \qquad n = \pm 1, \pm 2, ... \qquad (1)$$

One of the most important effects of equation (1) is the Aharonov-Bohm effect [5], where two-slit interference patterns of electrons show the action of the electromagnetic field in regions where it is absent. The clear interpretation of this phenomenon is by the non local character of the electromagnetic interaction [5].
Here, we adopt the point of view that the electromagnetic flux quantization is a natural consequence of the principles of quantum electrodynamics. It was suggested that the magnetic flux may be considered as an operator $\hat{\phi}_b$, whose canonical pair is the electric flux $\hat{\phi}_e$, and that both are linked in a ring by the commutation rule.

$$\left[\hat{\phi}_e, \hat{\phi}_b\right] = i\hbar \qquad (2)$$

Using this equation and applying a boundary condition on $\hat{\phi}_e$ that reflects the charge quantization one arrives at Eq. (I) [1].
In this paper, section 2 we introduce the electromagnetic field like a wave and we obtain equation (2). In section3, we show how to derive equation (2) from the quantization of Bohr model like a LC electric oscillator. In section 4 Energy transformation and conversion in hydrogen atom is discussed.

II. The Electromagnetic Flux Quantization

We will show here how equation (2) can be deduced from the principles of quantum eletrodynamics. Let us take the usual electromagnetic Lagrangian [1].

$$L = -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} \qquad (3)$$

With $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$
from which we immediately we obtain

$$\partial L / A_0 = 0 \text{ and } \partial L / A_k = F_{0k} = E_k \qquad (4)$$

The quantization of the temporal term will not concern us here. The spatial quantization rule is given by

$$\left[\hat{E}_i(\vec{x},t), \hat{A}_j(\vec{y},t)\right] = i\partial_{i,j}\partial^3(\vec{x}-\vec{y}) \qquad (5)$$

We define the electric flux and magnetic flux operators by

$$\hat{\phi}_{ew} = \int \hat{E}.dS_1 \qquad (6)$$

$$\hat{\phi}_{bw} = \int \vec{B}.dS_2 \quad (7)$$

Using equation (5), we obtain

$$\left[\hat{\phi}_{ew}, \hat{\phi}_{bw}\right] = \int\oint\left[\vec{E}_i(\vec{x},t), \vec{A}_j(\vec{y},t)\right] dS_1^i(\vec{x}) dl_2^j = i\hbar \quad (8)$$

$$\int\oint i\partial_{i,j}\partial^3(\vec{x}-\vec{y}) dS_1^i(\vec{x}) dl_2^j = i\hbar \quad (9)$$

where we adopted the repeated index sum rule convention.
Here, we claim that the electromagnetic flux quantization is suitable to study non local aspects of electrodynamics.

This Electromagnetic Flux Quantization (EFQ), corresponds to the configuration $\vec{E} \perp \vec{B}$, where the Poynting vector is nonzero so that the Bohr atom would be unstable if we include equation (8) in the original Bohr model. So what type of wave configuration is suitable for the Bohr model? In reference [6], the author consider that $\boldsymbol{E}_0 = \sqrt{\frac{\mu_0}{\varepsilon_0}}\boldsymbol{H}_0$ when $\vec{E} \perp \vec{B}$ which is wrong as we will see in the next section.

III. Formulation of the Problem of Bohr model like a L- C circuit

In the Bohr model, the radiant electromagnetic field is not considered because the electron to radiate energy fall spiral into the nucleus. Bohr thought that the atom was stable if electromagnetic wave should not radiate being the Poynting vector equal to zero, so he does not consider the electromagnetic wave on its own model, [7, 8]. The Born atom like a closed system can be analyzed in terms of capacitive and inductive elements, such as the classical L-C circuit. The energy stored in C is given by

$$U_C = \frac{Q^2}{2C} \quad (10)$$

where Q is the charge in the capacitor. But ,Q = $Q = \varepsilon\hat{\phi}_e$ where $\hat{\phi}_e$ is the electric flux in the capacitor, and ε is the dielectric constant of the capacitor medium. Therefore,

$$U_C = \frac{\varepsilon^2}{2C}\phi_e^2 \quad (11)$$

In the same way, the energy stored in L is given by

$$U_L = \frac{Li^2}{2} \quad (12)$$

However, from $\phi_b = Li$, where i is the current in the circuit, we have

$$U_L = \frac{1}{2L}\phi_b^2 \quad (13)$$

If we consider the photon energy interacting with the system electron-ion, (see section 2)

$$U_w = \frac{1}{2}\phi_{ew}^2 + \frac{1}{2}\phi_{bw}^2 + I \quad (14)$$

Where I is the interaction between the electron-ion Bohr system and the wave configuration (photon). Adding (11), (13) and (14), we get the total energy stored in the system.

$$H = \frac{\varepsilon^2}{2C}\phi_e^2 + \frac{1}{2L}\phi_b^2 + \frac{1}{2\varepsilon_0}\phi_{ew}^2 + \frac{1}{2\mu_0}\phi_{bw}^2 + I \quad (15)$$

Using the energy conservation we are led to

$$\frac{\varepsilon^2}{C}\phi_e\dot{\phi}_e + \frac{1}{L}\phi_b\dot{\phi}_b + \frac{1}{\varepsilon_0}\phi_{ew}\dot{\phi}_{ew} + \frac{1}{\mu_0}\phi_{bw}\dot{\phi}_{bw} + \dot{I} = 0 \quad (16)$$

which can be solved by

$$\dot{\phi}_{E+EW} = \frac{\partial H}{\partial\phi_{B+BW}} \quad \text{and} \quad \dot{\phi}_{B+BW} = -\frac{\partial H}{\partial\phi_{E+EW}} \quad (17)$$

This implies that the system described by (15) is Hamiltonian, and the new variables ϕ_{e+ew} and ϕ_{b+bw} form a canonical pair. Since they have, in this system, a role analogous to the (p, q) variables in classical mechanics. Therefore, the quantization of this system requires

$$\left[\hat{\phi}_{e+ew}, \hat{\phi}_{b+bw}\right] = i\hbar \qquad (18)$$

However for an electromagnetic waves the maximum electric field energy is $U_{ew} = \int \frac{1}{2}\varepsilon_0 \boldsymbol{E}_W^2 dv$ and the maximum magnetic energy is $U_{mw} = \int \frac{1}{2}\mu_0 \boldsymbol{H}_W^2 dv$ with the Maxwell condition $\vec{E} \perp \vec{H}$. Here, equation (18) is not satisfied because the terms

$$\frac{1}{\varepsilon_0}\phi_{ew}\dot{\phi}_{ew} + \frac{1}{\mu_0}\phi_{bw}\dot{\phi}_{bw} + \dot{I} \qquad (19)$$

are not constants so equation (18)is not satisfied.

In reference [6], the author Huang considers that $\boldsymbol{E}_0 = \sqrt{\frac{\mu_0}{\varepsilon_0}}\boldsymbol{H}_0$ when $\vec{E} \perp \vec{B}$, then equation (18) is not satisfied. According to our proposal Huang's solution is incorrect or wrong as we will see in the next section because the atom is unstable.

We intend to use and apply here the E.F.Q presented above before showing how it develops from quantum electrodynarnics. We think that its "deduction" from the quantization of an L-C circuit is convincing enough to carry out an immediate study of its applications.

From equation (10) we will build up an approach of the EFQ in which the states will be described in the electric flux representation. That means that the wave function will be given by

$$\psi = \left|\phi_{e+ew}\right\rangle \qquad (20)$$

and, in order to satisfy equation (10), we make

$$\hat{\phi}_{b+bw} = -i\hbar \frac{\partial}{\partial \phi_{e+ew}} \qquad (21)$$

where $\hat{\phi}_{e+ew}$ and $\hat{\phi}_{b+bw}$ are respectively the total electric and magnetic flux operators.

IV. Energy transformation and conversion in hydrogen atom

In quantum circuit [9], for a lossless LC quantum circuit, which is composed of an inductance L and a capacitance C , the charge q on the capacitance and the variable p satisfy the commutation relation $[Q, p] = i\hbar$, where p is given by $p(t) = L\frac{\partial q}{\partial t}$. Because the magnetic flux through the inductance $p(t) = L\frac{\partial q}{\partial t} = \phi$, and the voltage across the inductance (or the capacitance) $U = \frac{Q}{C}$, the commutation relation between U and Φ is: $C[U, \phi] = i\hbar$. (Here, Q , p ,U ,Φ are operators in quantum mechanics, C and L are constants). It means that any measurement on the magnetic flux Φ through a solenoid must be with a perturbation on its voltage U.

It should be pointed out that Eq. (18) to (21) are the foundation of our study.

The energy of the electromagnetic wave is

$$U_w = U_f = \int \frac{1}{2}(\varepsilon_0 \boldsymbol{E}_w^2 + \mu_0 \boldsymbol{H}_w^2) dv \qquad (22)$$

Which diverges when $\vec{E} \perp \vec{B}$, so equations (18) and (19) are not satisfied because the total energy electron-ion + electromagnetic energy +I is not a constant, and the electron emits radiation and spiraling into the nucleus.These two equations together must indicate a process of perfect periodically transformation of two forms of energy (kinetic energy $U_k = m_e u^2 / 2 = U_L$ and field energy $U_f = \frac{Q^2}{2C} = U_C$) inside the atom and the conservation of energy in the system

$$U_{total} = U_C = U_L \tag{23}$$

Recall the macroscopic harmonic LC oscillator where two forms of energy, the maximum field energy $U_C = \frac{Q^2}{2C}$ of the capacitor C (carrying a charge Q) and the maximum magnetic energy $U_L = \frac{L^2}{2L}$ of the inductor L, are mutually interchangeable ($U_{total} = U_C = U_L$) with a exchange periodic $T = 2\pi\sqrt{LC}$.
To satisfy equation (21), which is the Bohr equation, the microscopic photon or electromagnetic wave must be considered as a stationary wave in instanton configuration [8, 9].
The maximum field energy

$$U_{fw} = \int \frac{1}{2}\varepsilon_0 \boldsymbol{E}_w^2 dv \tag{24}$$

and the maximum magnetic energy

$$U_{mw} = \int \frac{1}{2}\mu_0 \boldsymbol{H}_w^2 dv \tag{25}$$

must satisfy

$$U_{w_{total}} = U_{fw} + U_{kw} = 0, \tag{26}$$

so the interaction must be

$$I = 0 \tag{27}$$

and

$$\boldsymbol{E}_W = i\sqrt{\frac{\mu_0}{\varepsilon_0}}\boldsymbol{H}_W . \tag{28}$$

In this configuration with $\boldsymbol{E}_W$ II $\boldsymbol{H}_W$ the Pointing vector is $\boldsymbol{P}_W = \boldsymbol{E}_W \times \boldsymbol{H}_W = 0$, so the Bohr atom is stable.
Following [9], in this reference we show that the quantization appears for the Bohr atom with n as the principal quantization number.
Based on the above energy relationship for three totally different systems and the requirement of the electromagnetic interaction (by exchanging photon) between electron and nuclei, we assure that the kinetic energy of electron, Eq. (21) is a kind of magnetic energy and the hydrogen atom is a natural microscopic LC oscillator [10].
The Bohr's model seems newly reconciled with quantum mechanics and yields surprisingly accurate predictions for hydrogen and other small molecules. Even today the Bohr model has valid roles in describing highly exited Ryberg atoms cavity quantum electrodynamics and quasi-Ridberg states in graphene. [11]

Conclusion

The quantum approach to the electric and magnetic fluxes was reconsidered. These quantities was treated as operators, and we discuss how this approach takes into account of the magnetic flux quantization in the Bohr atom. We discuss the derivation of the electromagnetic flux quantization. We also show that this quantization rule can be found from an elementary analysis of a Bohr electric oscillator like an L-C circuit including the electromagnetic wave like an instanton configuration.

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