Some New Prime Graphs

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ABSTRACT : *A Graph G with n vertices is said to admit prime labeling if its vertices can be labeled with distinct positive integers not exceeding n such that the labels of each pair of adjacent vertices are relatively prime. A graph G which admits prime labeling is called a prime graph. In this paper we investigate the existence of prime labeling of some graphs related to cycle* C_n, *wheel* W_n,*crown* C_n^*,*Helm*H_n*and Gear graph*G_n,*Stars*$_n$, *Friendship graph* T_n,*prism* D_n *and Butterfly graph* $B_{n,m}$ *.We discuss prime labeling in the context of the graph operation namely duplication.*

Keywords:*Graph Labeling, Prime Labeling, Duplication, Prime Graph.*

I. INTRODUCTION

We begin with simple, finite, connected and undirected graph G (V,E) with p vertices and q edges.The set of vertices adjacent to a vertex u of G is denoted by N(u).For notations and terminology we refer to Bondy and Murthy[1].

The notion of prime labeling was introduced by Roger Entringer and was discussed in a paper by Tout[6]. Two integers a and b are said to be relatively prime if their greatest common divisor is 1. Relatively prime numbers play an important role in both analytic and algebraic number theory. Many researchers have studied prime graph.Fu. H [3] has proved that the path P_n onn vertices is a prime graph. Deretsky el al [2] haveprove that the cycle C_n on n vertices is a prime graph. Around 1980 rugerEtringer conjectured that all trees have prime labeling which is not settled till today.

The prime labeling for planer grid is investigated by Sundaram et al [5] , Lee.S.al [4] has proved that the Wheel W_n is a prime graph if and only if n is even.

Definition 1.1[7] Duplication of a vertex v_k by a new edge e = $v_k^{'} v_k^{''}$ in a graph G produces a new graph G' such that N($v_k^{'}$) ∩ N($v_k^{''}$)= v_k

Definition 1.2 The graph obtained by duplicating all the vertices by edges of a graph G is called duplication of G

Definition 1.3 The crown graph C_n^* is obtained from a cycle C_n by attaching a pendent edge at each vertex of the n-cycle.

Definition 1.4 The HelmH_n is a graph obtained from a Wheel by attaching a pendent edge at each vertex of the n-cycle.

Definition 1.5 The gear graph$G_n$is, the graph obtained from wheel $W_n = C_n + K_1$by subdividing each edge incident with the apex vertex once.

Definition 1.6 TheFriendship graph T_n is set of n triangles having a common central vertex.

Definition 1.7 The Prism D_n is a Graph obtained from the cycleC_n(= $v_1, v_2, \ldots, v_n$)by attaching n-3 chords$v_1 v_3, v_1 v_4, \ldots, v_1 v_{n-2}$.

Definition 1.8 The Butterfly Graph $B_{n,m}$ is,the Graph obtained from two copies ofC_n having one vertex in common, and by attaching m pendent edges tothe common vertex of two cycles.

In this paper we proved that the graphs obtained by duplication of all the alternate vertices by edges in$C_n$and wheel W_n if n is even , duplicatingall the rim vertices byedges in crown C_n^*,Helm H_n andGear graphG_n, duplicating all the alternate vertices by edges in star $S_n = K_{1,n}$ and Friendship graph $T_n$and prism D_n,duplicating all the vertices of C_n by edges in Butterfly graph $B_{n,m}$, m ≥ 2n-2 are all prime graphs.

II. MAIN RESULTS

Theorem 2.1

The graph obtained by duplicating all the alternate vertices by edges inC_n is a prime graph , if n is even.

Proof.

Let$V(C_n) = \{u_i \ / \ 1 \le i \le n\}$

$E(C_n) = \{u_i\, u_{i+1} \,/\, 1 \le i \le n\} \cup \{u_n u_1\}$

Let G be the graph obtained by duplicatingall the alternate vertices by edges inC_n and let the new edges be$u_1^{'}u_1^{"}, u_3^{'}u_3^{"}, \dots, u_{n-1}^{'}u_{n-1}^{"}$by duplicating the alternate vertices $u_1, u_3, \dots. u_{n-1}$respectively,

Then$V(G) = \{u_i \ / \ 1 \le i \le n\} \cup \{u_i^{'}, u_i^{"} \ /1 \le i \le n, i \text{ is odd}\}$

$$E(G) = \{u_i\, u_{i+1} \ /1 \le i \le n-1\} \cup \{u_i u_i^{'}, u_i u_i^{"}, u_i^{'} u_i^{"} \ / \ 1 \le i \le n, i \text{ is odd}\} \cup \{u_n u_1\}$$

$|V(G)| = 2n, \quad |E(G)| = 5n/2$

Define a labeling $f : V(G) \to \{1,2,3, \dots, 2n\}$as follows.

Let $f(u_1) = 1$

$$f(u_i) = 2i \qquad if\ i\ is\ even, \quad 2 \le i \le n,$$
$$f(u_i) = 2i - 1 \qquad if\ i\ is\ odd, \quad 3 \le i \le n-1 \quad and \quad i \not\equiv 2(mod\ 3)$$
$$f(u_i) = 2i + 1, \quad if\ i\ is\ odd, \quad 1 \le i \le n-1 \quad and \quad i \equiv 2(mod\ 3) f(u_i^{"})$$
$$= 2i - 1, \quad if\ i\ is\ odd, \quad 1 \le i \le n-1 \quad and \quad i \equiv 2(mod\ 3)$$
$$f(u_i^{'}) = 2i, \qquad if\ i\ is\ odd, \quad 1 \le i \le n-1$$
$$f(u_i^{"}) = 2i + 1, \quad if\ i\ is\ odd, \quad 1 \le i \le n-1 \quad and \quad i \not\equiv 2(mod\ 3)$$

Since $f(u_1) = 1$

$\gcd(f(u_1), f(u_2))$=1, $\gcd\left(f(u_1), f(u_1^{'})\right)$=1,

$\gcd\left(f(u_1), f(u_1^{"})\right)$= 1, $\gcd(f(u_n), f(u_1))$ = 1.

$$\gcd(f(u_i), f(u_{i+1})) = \gcd(2i, 2(i+1) - 1) = \gcd(2i, 2i+1)$$

$= 1, \qquad i\ is\ even, \qquad for\ 2 \le i \le n,\ i \not\equiv 2(mod\ 3)$If i is odd .

$\gcd(f(u_i), f(u_{i+1})) = \gcd(2i - 1, 2(i+1))$

$= \gcd(2i-1, 2i+2) \quad = 1 \quad for\ 3 \le i \le n, i \not\equiv 2(mod\ 3)$

as among these two numbers one is odd and other is even and their difference is 3 and they are not multiples of 3

$\gcd(f(u_{i-1}), f(u_i)) = \gcd(2(i-1), 2i+1)$

$= \gcd(2i - 2, 2i + 1) \quad = 1 \quad for\ 3 \le i \le n, i \equiv 2(mod\ 3)$

as among these two numbers one is even and other is odd their difference is 3 . And they are not multiples of 3

$\gcd(f(u_i), f(u_{i+1})) = \gcd(2i + 1, 2i + 2) \quad = 1 \quad for\ 3 \le i \le n,\ i \equiv 2(mod\ 3)$

$\gcd\left(fu_i), f(u_i^{'})\right)$= $\gcd(2i - 1, 2i)$= 1 $for\ 3 \le i \le n,\ i \equiv 2(mod\ 3)$

as these two number are consecutive integers

$\gcd\left(f(u_i), f(u_i^{"})\right)$= $\gcd(2i - 1, 2i + 1)$=1 $for\ 3 \le i \le n,\ i \not\equiv 2(mod\ 3)$

$\gcd\left(f(u_i), f(u_i^{"})\right)$= $\gcd(2i + 1, 2i - 1)$= 1 $for\ 3 \le i \le$ n, $i \equiv 2(mod\ 3)$

as these two number are odd consecutive integers

$\gcd\left(f(u_i^{'}), f(u_i^{"})\right)$= $\gcd(2i, 2i + 1)$= 1 $for\ 1 \le i \le$ n, $i \not\equiv 2(mod\ 3)$

$\gcd\left(f(u_i^{'}), f(u_i^{"})\right)$= $\gcd(2i, 2i - 1)$= 1 $for\ 1 \le i \le$ n, $i \equiv 2(mod\ 3)$

as these two number are consecutive integers

Thus$f$ is a prime labeling.

HenceG is a prime graph.

Theorem 2.2

The graph obtained by duplicating all the alternate rim vertices by edges in Wheel W_n is a prime graph. If n is even and $n \not\equiv 1(mod\ 3)$.

Proof.

Let$V(W_n) = \{ c, u_i \ / \ 1 \le i \le n\}$

$E(W_n) = \{ cu_i \ / 1 \le i \le n \} \cup \{u_i u_{i+1} / 1 \le i \le n-1 \} \cup \{ u_n u_1 \}$

Let G be the graph obtained by duplicating all the alternate rim vertices by edges in Wheel W_n and let the new edges be $u_1^{'}u_1^{"}, u_3^{'}u_3^{"}, \ldots, u_{n-1}^{'}u_{n-1}^{"}$by duplicating the vertices $u_1, u_3, \ldots . u_{n-1}$respectively,

$$V(G) = \{ c, u_i \ / \ 1 \le i \le n\} \cup \{u_i^{'}, u_i^{"} / 1 \le i \le n-1, i \text{ is odd}\}$$

$$E(G) = \{cu_i/1 \le i \le n\} \cup \{u_i u_{i+1}/1 \le i \le n-1\} \cup \{u_i u_i^{'}, u_i u_i^{"}, u_i^{'}u_i^{"} \ /1 \le i \le n, i is odd\} \cup \{ u_n u_1\}$$

$$|V(G)| = 2n+1, \qquad |E(G)| = 7n/2$$

Define a labeling $f : V(G) \to \{1,2,3, \ldots, 2n+1\}$as follows

Let $f(c) = 1, f(u_1) = 2n+1$

$$f(u_i) = \begin{cases} 2i, & for \ 2 \le i \le n, \quad i \text{ is even} \\ 2i-1, & for \ 3 \le i \le n-1, \quad i \text{ is odd}, \end{cases} \quad i \not\equiv 2(mod\ 3)$$

$$f(u_i) = 2i+1, \quad for \ 1 \le i \le n-1, \quad i \text{ is odd}, \ i \equiv 2(mod\ 3)$$

$$f(u_i^{'}) = 2i, \quad for \ 1 \le i \le n-1, \quad i \text{ is odd}$$

$$f(u_i^{"}) = 2i+1, \quad for \ 1 \le i \le n-1, \quad i \text{ is odd}, \ i \not\equiv 2(mod\ 3)$$

$$f(u_i^{"}) = 2i-1, \quad for \ 1 \le i \le n-1, \quad i \text{ is odd}, \ i \equiv 2(mod\ 3)$$

Since$f(c) = 1$

$\gcd(f(c), f(u_i)) = 1, \qquad for \ 1 \le i \le n$

$\gcd(f(u_1), f(u_1^{'}))= \gcd(2n+1, 2) = 1$

$\gcd(f(u_1), f(u_i^{"}))= \gcd(2n+1, 3) = 1, \ for \ n \not\equiv 1(mod\ 3)$

Since$n \not\equiv 1(mod\ 3), \ 2n+1 \not\equiv 0(mod\ 3)$

Therefore $\gcd(f(u_1), f(u_i^{"}))$= (2n+1,3)= 1,

Similar to previous theorem for all other pair of adjacent vertices gcd =1

Thus f is a prime labeling.

Hence G is a prime graph.

Theorem 2.3

The graph obtained by duplicating all the rim vertices by edges inCrown $C_n^*$is a prime graph.

Proof:

Let$V(C_n^*) = \{u_i, v_i \ / \ 1 \le i \le n\}$

$E(C_n^*)=\{u_i u_{i+1} / 1 \le i \le n\} \cup \{u_i v_i / 1 \le i \le n\} \cup \{u_n u_1\}$

Let G be the graph obtained by duplicating all the rim vertices by edges inCrown $C_n^*$and let the new edges be $u_1^{'}u_1^{"}, u_2^{'}u_2^{"}, \ldots, u_n^{'}u_n^{"}$by duplicating the vertices $u_1, u_2, \ldots . u_n$respectively,

Then,

$$V(G) = \{ u_i, v_i, u_i^{'}, u_i^{"} \ / \ 1 \le i \le n\}$$

$$E(G) = \{u_i u_{i+1}/1 \le i \le n-1\} \cup \{u_i v_i, u_i u_i^{'}, u_i^{'}u_i^{"}, u_i u_i^{"}/1 \le i \le n\} \cup \{ u_n u_1\} |V(G)| = 4n, \quad |E(G)| = 5n.$$

Define a labeling $f : V(G) \to \{1,2,3, \ldots, 4n\}$as follows

Let $f(u_1) = 1, f(u_1^{'}) = 2, f(u_1^{"}) = 3$ and$f(v_1) = 4$.

$f(v_i) = 4i, \qquad for \ 2 \le i \le n$

$f(u_i) = 4i-1, \qquad for \ 2 \le i \le n,$

$f(u_i^{'}) = 4i-3, \qquad for \ 2 \le i \le n,$

$f(u_i^{"}) = 4i-2, \qquad for \ 2 \le i \le n,$

Since $f(u_1) = 1$

$\gcd(f(u_1), f(u_2))$= 1, $\gcd(f(u_1), f(u_1^{'}))$= 1,$\gcd(f(u_1), f(u_1^{"}))$= 1,

$\gcd(f(u_1), f(u_n))$=1, $\gcd(f(u_1), f(v_1))$= 1.

Then

$\gcd(f(u_i), f(u_{i+1}))= \gcd(4i-1), 4(i+1)-1)$

$= \gcd(4i-1, 4i+3)$= 1for $2 \le i \le n$

$\gcd\left(f(u_i), f(u_i^{'})\right)$= $\gcd(4i-1,4i-3)$= 1for 2$\leq i \leq n$
as these two numbers are odd and their differences are 4,2 respectively.
$\gcd\left(f(u_i), f(u_i^{"})\right)$= $\gcd(4i-1,4i-2)$= 1for 2$\leq i \leq n$
$\gcd(f(u_i), f(v_i))$= $\gcd(4i-1, 4i)$= 1for 2$\leq i \leq n$
as they are consecutive integers
Thus$f$ is a prime labeling.
Hence G is a prime graph.

Theorem 2.4

The graph obtained by duplicating all the rim vertices by edges in Helm $H_n$is a prime graph..If $n \not\equiv 4(mod\ 5)$

Proof.
Let $V(H_n) = \{c, u_i, v_i \ / \ 1 \leq i \leq n\}$
$E(H_n)$ =$\{cu_i, u_iv_i \ /1 \leq i \leq n\} \cup \{u_i\, u_{i+1} \ / \ 1 \leq i \leq n-1\} \cup \{u_n u_1\}$
Let G be the graph obtained by duplicating all the rim vertices by edgesin Helm $H_n$and let the new edges be $u_1^{'}u_1^{"}, u_2^{'}u_2^{"}, \ldots, u_n^{'}u_n^{"}$by duplicating the rim vertices $u_1, u_2, \ldots. u_n$respectively,
Then,
$V(G) = \{c, u_i, v_i, u_i^{'}, u_i^{"} \ / \ 1 \leq i \leq n\}$$E(G) = \{cu_i, u_iv_i, u_iu_i^{'}, u_iu_i^{"}, u_i^{'}u_i^{"} \ /1 \leq i \leq n\} \cup \{u_iu_{i+1}/1 \leq i \leq n-$ *1∪{ unu1}. VG=4n+1, EG=6n,*
Define a labeling $f : V(G) \rightarrow \{1,2,3, \ldots, 4n+1\}$as follows
Let$f(c) = 1$, $f(u_1) = 5, f(u_1^{'}) = 3, f(u_1^{"}) = 4$, and$f(v_1) = 2$.

$f(u_i) = 4i-1$, $\quad$ *for* $2 \leq i \leq n$,
$f(u_i^{'}) = 4i$, $\quad$ *for* $2 \leq i \leq n$,
$f(u_i^{"}) = 4i+1$, $\quad$ *for* $2 \leq i \leq n$,
$f(v_i) = 4i-2$, $\quad$ *for* $2 \leq i \leq n$

Since $f(c) = 1$
$\gcd(f(c), f(u_i))$= 1
$\gcd(f(u_1), f(u_1^{'}))$=gcd(5,3) = 1
$\gcd\left(f(u_1), f(u_1^{"})\right)$ = gcd(5,4)= 1
$\gcd(f(u_1), f(v_1))$= gcd(5,2)= 1
$\gcd(f(u_1), f(u_2))$ = gcd(5,7)=1
$\gcd(f(u_n), f(u_1))$ = gcd(4n − 1,5)=1
Since $n \not\equiv 4(mod\ 5)$ and 4n-1 is not a multiple of 5
$\gcd(f(u_i), f(u_{i+1}))$= $\gcd(4i-1), 4(i+1)-1)$
= $\gcd(4i-1, 4i+3)$= 1for 2$\leq i \leq n$
$\gcd\left(f(u_i), f(u_i^{"})\right)$= $\gcd(4i-1,4i+1)$= 1for 2$\leq i \leq n$
as these two numbers are odd and also their differences are 4,2 respectively.
$\gcd(f(u_i), f(u_i^{'}))$= $\gcd(4i-1,4i)$= 1for 2$\leq i \leq n$
$\gcd(f(u_i), f(v_i))$= $\gcd(4i-1,4i-2)$= 1for 2$\leq i \leq n$
as they are consecutive integers
Thus$f$ is a prime labeling.
Hence G is a prime graph.

Theorem 2.5

The graph obtained by duplicating all the rim vertices by edges in Gear$G_n$is a prime graph..If $n \not\equiv 4(mod\ 5)$

Proof.
Let $V(G_n)$= $\{c, u_i, v_i \ / \ 1 \leq i \leq n\}$
$E(G_n)$ =$\{cv_i, v_iu_i \ /1 \leq i \leq n\} \cup \{u_i\, u_{i+1} \ / \ 1 \leq i \leq n-1\} \cup \{u_n u_1\}$
Let G be the graph obtained by duplicating all the rim vertices by edgesin Gear $G_n$and let the new edges be $u_1^{'}u_1^{"}, u_2^{'}u_2^{"}, \ldots, u_n^{'}u_n^{"}$by duplicating the rim vertices $u_1, u_2, \ldots. u_n$respectively,
Then,

$V(G) = \{c, u_i, u_i^{'}, u_i^{"}, v_i \ / \ 1 \le i \le n\} E(G) = \{cv_i, v_iu_i, u_iu_i^{'}, u_iu_i^{"}, u_i^{'}u_i^{"} \ /1 \le i \le n\} \cup \{u_iu_{i+1}/1 \le i \le n -$ *1∪{ unu1}.VG=4n+1, EG=6n.*

Define a labeling $f : V(G) \to \{1,2,3, \dots, 4n + 1\}$as follows

Let$f(c) = 1, f(u_1) = 5, f(u_1^{'}) = 3, f(u_1^{"}) = 4,$ and $f(v_1) = 2.$

$$f(u_i) = 4i - 1, \quad for\ 2 \le i \le n,$$
$$f(u_i^{'}) = 4i, \quad for\ 2 \le i \le n,$$
$$f(u_i^{"}) = 4i + 1, \quad for\ 2 \le i \le n,$$
$$f(v_i) = 4i - 2, \quad for\ 2 \le i \le n$$

Since $f(c) = 1$

$\gcd(f(c), f(v_i)) = 1, \quad for\ 1 \le i \le n$

Similar to the previous theorem we can show that for all other pair of adjacent vertices g.c.d is 1

Thus$f$ is a prime labeling.

Hence G is a prime graph.

Theorem 2.6

The graph obtained by duplicating the centre vertex andall the alternate vertices by edges in Star $S_n = K_{1,n}$ is a prime graph.. If n is even

Proof.

Let$V(S_n) = \{c, u_i \ / \ 1 \le i \le n\}$

$E(S_n) = \{cu_i /1 \le i \le n\}$

Let G be the graph obtained by duplicating the centre vertex and all the alternate vertices by edgesin Star $S_n = K_{1,n}$and let the new edges be $c^{'}c^{"}, u_1^{'}u_1^{"}, u_3^{'}u_3^{"}, \dots, u_{n-1}^{'}u_{n-1}^{"}$by duplicating the vertices $c, u_1, u_3, \dots. u_{n-1}$respectively,

Then,

$V(G) = \{c, c^{'}, c^{"}, u_i \ / \ 1 \le i \le n\} \cup \{u_i^{'}, u_i^{"}/1 \le i \le n, i\ is\ odd\} E(G) = \{cu_i, cc^{'}, cc^{"}, c^{'}c^{"}/1 \le i \le n\} \cup \{u_iu_i^{'}, u_iu_i^{"}, u_i^{'}u_i^{"}/1 \le i \le n, i\ is\ odd\}.$

$|V(G)| = 2n + 3, \ |E(G)| = \frac{5n}{2}+3$

Define a labeling $f : V(G) \to \{1,2,3, \dots, 2n + 3\}$as follows

Let$f(c) = 1, \ f(c^{'}) = 2n + 2, \ f(c^{"}) = 2n + 3. f(u_i) = \begin{cases} 2i + 1, & for\ 1 \le i \le n, \quad i\ is\ odd \\ 2i - 2, & for\ 2 \le i \le n, \quad i\ is\ even, \end{cases}$

$f(u_i^{'}) = 2i + 2, \quad for\ 1 \le i \le n, \quad i\ is\ odd$

$f(u_i^{"}) = 2i + 3, \quad for\ 1 \le i \le n, \quad i\ is\ odd$

Since $f(c) = 1$

$\gcd(f(c), f(u_i)) = 1, \quad for\ 1 \le i \le n$

$\gcd(f(c), f(c^{'})) = 1, \quad \gcd(f(c), f(c^{"})) = 1$

$\gcd(f(c^{'}), f(c^{"})) = \gcd(2n + 2, 2n + 3) = 1$

$\gcd(f(u_i), f(u_i^{'})) = \gcd(2i + 1, 2i + 2) = 1$for $1 \le i \le n$

$\gcd(f(u_i^{'}), f(u_i^{"})) = \gcd(2i + 2, 2i + 3) = 1$for $1 \le i \le n$

$\gcd(f(u_i), f(u_i^{"})) = \gcd(2i + 1, 2i + 3) = 1$for $1 \le i \le n$

as they are consecutive odd integers

Thus$f$ is a prime labeling.

Hence G is a prime graph.

Theorem 2.7

The graph obtained by duplicating all the alternate vertices by edges in Friendship graph T_n, except centre,is a prime graph.

Proof.

Let $V(T_n) = \{c, u_i \ / \ 1 \le i \le 2n\}$

$E(T_n) = \{cu_i /1 \le i \le 2n\} \cup \{u_i u_{i+1} / 1 \le i \le 2n - 1, \ i$ is odd$\}$.

Let G be the graph obtained by duplicating all the alternate vertices by edgesin $T_n$and let the new edges be $u_1^{'}u_1^{"}, u_3^{'}u_3^{"}, \dots, u_{2n-1}^{'}u_{2n-1}^{"}$by duplicating the vertices$u_1, u_3, \dots. u_{2n-1}$respectively,

Then

$V(G) = \{c, u_i, u_i^{'}, u_i^{''} \;/\; 1 \le i \le 2n\}$
$E(G)$=$\{cu_i \;/ 1 \le i \le 2n\} \cup \{u_i u_i^{'}, u_i u_i^{''}, u_i^{'} u_i^{''}, u_i u_{i+1} \;/$ 1≤i≤2n-1,*i*is odd}
$|V(G)| = 4n + 1, \quad |E(G)| = 6n,$
Define a labeling $f : V(G) \to \{1,2,3, \ldots, 4n + 1\}$as follows
Let $f(c) = 1.$

$$f(u_i) = \begin{cases} 2i + 1, & for \;\; 1 \le i \le 2n - 1, \quad i \; is \; odd \\ 2i - 2, & for \;\; 2 \le i \le 2n, \quad i \; is \; even \end{cases}$$

$$f(u_i^{'}) = 2i + 2, \quad for \;\; 1 \le i \le 2n - 1, \quad i \; is \; odd$$
$$f(u_i^{''}) = 2i + 3, \quad for \;\; 1 \le i \le 2n - 1, \quad i \; is \; odd$$

Since $f(c) = 1$
$\gcd(f(c), f(u_i))$= 1, $\quad for \;\; 1 \le i \le 2n - 1, \quad i \; is \; odd$
$\gcd(f(c), f(u_i))$= 1, $\quad for \;\; 2 \le i \le 2n, \quad i \; is \; even$
If *i* is odd
$\gcd(f(u_i), f(u_{i+1}))$= $\gcd(2i + 1, 2(i + 1) - 2)$
= $\gcd(2i + 1, 2i) \quad = 1 \quad$ for $\; 1 \le i \le 2n - 1$
$\gcd(f(u_i), f(u_i^{'}))$= $\gcd(2i + 1, 2i + 2) \quad = 1 \quad$ for $\; 1 \le i \le 2n - 1$
$\gcd\left(f(u_i), f\left(u_i^{''}\right)\right)$= $\gcd(2i + 1, 2i + 3)$= 1for $\; 1 \le i \le 2n - 1$
.$\gcd\left(f(u_i^{'}), f\left(u_i^{''}\right)\right)$= $\gcd(2i + 2, 2i + 3)$= 1for $\; 1 \le i \le 2n - 1$
Thus *f*is a prime labeling
Hence G is a prime graph.

Theorem 2.8
The graph obtained by duplicating all the alternate vertices by edges in Prism $D_n$is a prime graph.Where n is even.

Proof.
Let$V(D_n) = \{u_i \;/\; 1 \le i \le n\}$
$E(D_n) = \{u_i u_{i+1} \,/\, 1 \le i \le n - 1\} \cup \{u_n u_1\} \cup \{u_1 u_i \,/\, 3 \le i \le n - 1\}$
Let G be the graph obtained by duplicating all the alternate vertices by edges in Prime D_n and let the new edges be $u_1^{'} u_1^{''}, u_3^{'} u_3^{''}, \ldots, u_{n-1}^{'} u_{n-1}^{''}$by duplicating the vertices $u_1, u_3, \ldots . u_{n-1}$respectively,
$V(G) = \{u_i \;/\; 1 \le i \le n\} \cup \{u_i^{'}, u_i^{''} \;/\; 1 \le i \le n - 1, i \; is \; odd\} E(G) =$
$\{u_i u_{i+1} / 1 \le i \le n - 1\} \cup \{u_i u_i^{'}, u_i u_i^{''}, u_i^{'} u_i^{''} / 1 \le i \le n - 1, i is odd\} \cup \{u_n u_1\} \cup \{u_1 u_i / 3 \le i \le n - 1\}.$
$|V(G)| = 3n, \; |E(G)| = (7n - 6)/2.$
Let $f(u_1) = 1.$

$$f(u_i) = \begin{cases} 2i, & for \;\; 2 \le i \le n, \quad i \; is \; even \\ 2i - 1, & for \; 3 \le i \le n - 1, \quad i \; is \; odd, \quad i \not\equiv 2 (mod \; 3), \end{cases}$$

$$f(u_i) = 2i + 1, \quad for \;\; 3 \le i \le n - 1, \;\; i \; is \; odd, \;\; i \equiv 2 (mod \; 3)$$
$$f(u_i^{'}) = 2i, \quad for \;\; 1 \le i \le n, \quad i \; is \; odd$$
$$f(u_i^{''}) = 2i + 1, \quad for \;\; 1 \le i \le n, \quad i \; is \; odd, \;\; i \not\equiv 2 (mod \; 3)$$
$$f(u_i^{''}) = 2i - 1, \quad for \;\; 1 \le i \le n, \quad i \; is \; odd, \;\; i \equiv 2 (mod \; 3)$$

Since $f(u_1) \;\; = 1$
$\gcd(f(u_1), f(u_i))$= 1for3 ≤*i*≤ n-1
Similar to the theorem 2.1 we canshow that for all other pair of adjacent vertices
g.c.dis 1
Thus *f*isa prime labeling
Hence G is a prime graph.

Theorem 2.9
The graph obtained by duplicating all the vertices of the cycles by edgesin Butterfly graph$B_{n,m}$ is a prime graph. If m ≥ 2n-2
Proof.
Let$V(B_{n,m}) = \{u_i \;/\; 1 \le i \le 2n - 1\} \cup \{v_i / \;\; 1 \le i \le m\} E(B_{n,m})$=$\{u_i u_{i+1} \,/ 1 \le i \le n - 1\} \cup \{u_i u_{i+1} \,/\, (n + 1) \le i \le 2n - 2\} \cup \{u_n u_1 \;, u_1 u_{n+1}, u_{2n-1} u_1\} \;\; \cup \{u_1 v_i \,/ 1 \le i \le m\}$

Case (i). If m = 2n -2

Let G be the graph obtained by duplicating all the vertices of two copies of C_n by edgesin $B_{n,m}$ and let the new edges be $u_1^{'}u_1^{"}, u_2^{'}u_2^{"}, \dots, u_{2n-1}^{'}u_{2n-1}^{"}$by duplicating the vertices $u_1, u_2, \dots . u_{2n-1}$respectively,

Then,

$V(G) = \{u_i, u_i^{'}, u_i^{"} \ / \ 1 \leq i \leq 2n-1\} \cup \{v_i \ / \ 1 \leq i \leq m\}$ $E(G) = \{u_i u_{i+1}/1 \leq i \leq n-1\} \cup \{u_i u_{i+1}/ \ n+1 \leq i \leq 2n-2\} \cup \{u_i u_i^{'}, u_i u_i^{"}, u_i^{'} u_i^{"}/1 \leq i \leq 2n-1\} \cup \{u_n u_1, u_1 u_{n+1}, u_{2n-1} u_1\} \cup \{u_1 v_i/1 \leq i \leq m\}$.

$|V(G)| = 6n - 3 + m, \quad |E(G)| = 8n - 3 + m.$

Define a labeling $f : V(G) \to \{1,2,3, \dots, 6n-3+m\}$as follows

Let$f(u_1) = 1$

$f(u_i) = 4i - 3, \quad for \ 1 \leq i \leq n \ and \ n+1 \leq i \leq 2n-1$

$f(u_i^{'}) = 4i - 2, \quad for \ 1 \leq i \leq 2n-1,$

$f(u_i^{"}) = 4i - 1, \quad for \ 1 \leq i \leq 2n-1,$

$f(v_i) = 4i, \quad for \ 1 \leq i \leq m \ (= 2n-2)$

Since $f(u_1) = 1$

gcd$(f(u_1), f(v_i))$= 1 $\quad for \ 1 \leq i \leq m.$

gcd$(f(u_1), f(u_n))$= 1 gcd$(f(u_1), f(u_{n+1}))$= 1 gcd$(f(u_1), f(u_{2n-1}))$= 1

gcd$(f(u_i), f(u_{i+1}))$= gcd$(4i-3), 4(i+1)-3)$

= gcd$(4i-3, 4i+1)$= 1for 1≤ $i \leq n$-1 and n+1≤ $i \leq 2n$-2

gcd $\left(f(u_i), f(u_i^{"})\right)$= gcd$(4i-3, 4i-1)$= 1for 1≤ $i \leq$ n-1 and n+1≤ $i \leq 2n$-2

as these two numbers are odd and their differences are 4,2 respectively.

gcd$(f(u_i), f(u_i^{'}))$= gcd$(4i-3, 4i-2)$= 1for 1≤ $i \leq$n-1 and n+1≤ $i \leq 2n$-2

Then f is a prime labeling.

Case (ii).If m > 2n -2

Then the same labeling given in case (i) can be given for all the vertices up to m=2n-2. For all other vertices give consecutive labelsso that the resulting labeling is prime.

Thus f is a prime labeling

Hence G is a prime graph.

Examples

Illustration 2.1

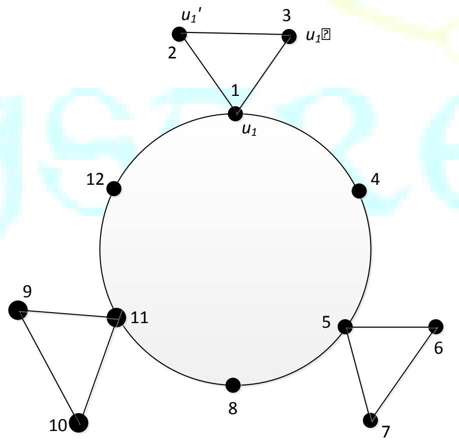

Figure1. Prime labeling of duplication of all the alternate vertices by edges in C_6

Illustration 2.2

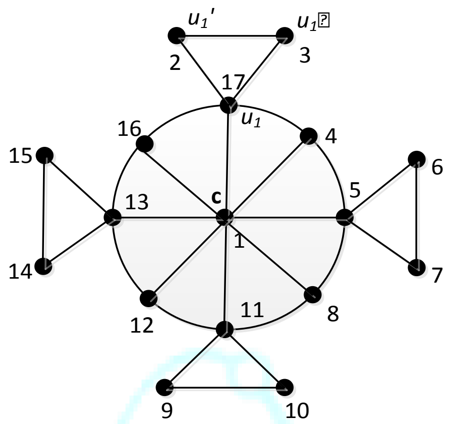

Figure1. Prime labeling of duplication of all the alternate rim vertices by edges in C_8 .

Illustration 2.3

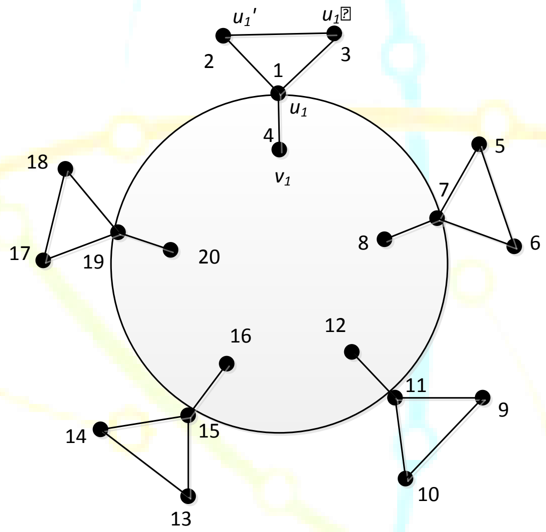

Figure1. Prime labeling of duplication of all the rim vertices by edges in Crown C_5^*

Illustration 2.4

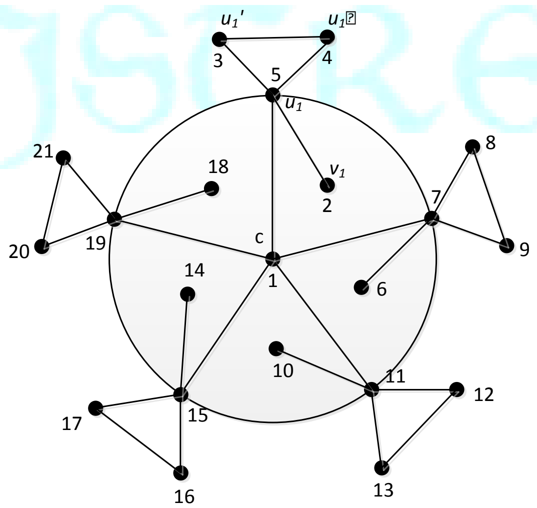

Figure1. Prime labeling of duplication of all the rim vertices by edges in HelmH_5

Illustration 2.5

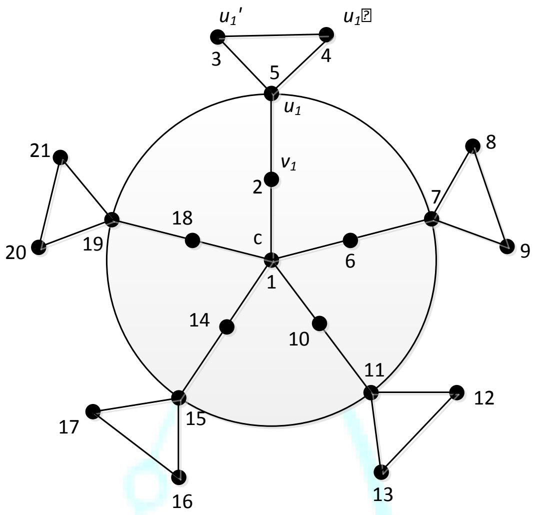

Figure1. Prime labeling of duplication of all the rim vertices by edges in Gear graph G_5.

Illustration 2.7

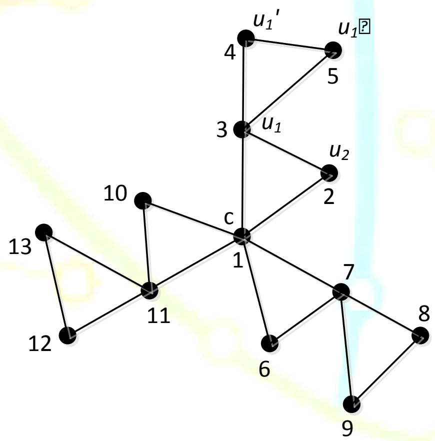

Figure1. Prime labeling of duplication of all the alternate vertices by edges in

Friendship graph T_3 , except centre .

Illustration 2.8

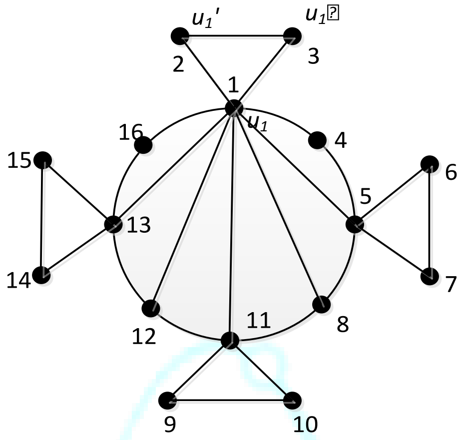

Figure1. Prime labeling of duplication of all the alternate vertices by edges inPrism D_8

Illustration 2.9

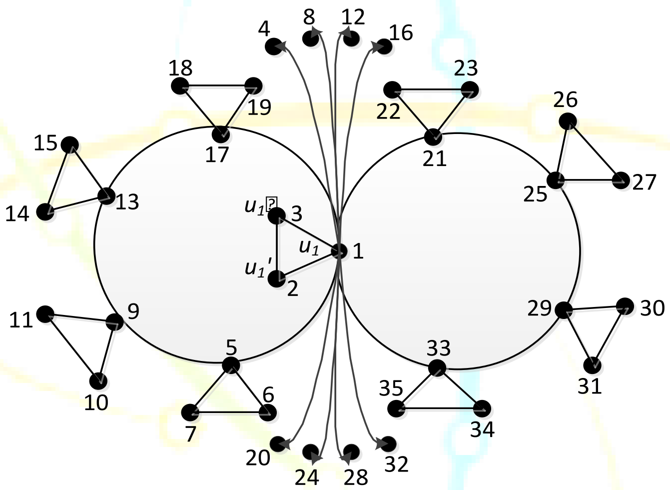

Figure1. Prime labeling of duplication of all the vertices of two cycles C_5 by edges in Butterfly graph $B_{5,8}$.

III. CONCLUSION

Here we investigate Nine corresponding results on prime labeling analogues work can be carried out for other families also.

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