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Extreme Rainfall Modeling Using Spatial Extreme Value with Max-Stable Processes Models of Smith, Brown-Resnick, and Schlather

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***Abstract**: Extreme weather is a phenomenon of an extreme nature and short-term scale, which can certainly have a serious impact on various life activities. One of the statistical method developed to analyze extreme events is Extreme Value Theory (EVT). The EVT method was developed for univariate cases with extreme event in one variable only. In some cases such as extreme rainfall observed in several locations, the EVT method cannot be used, therefore another method was developed, namely Spatial Extreme Value (SEV) with Max-Stable Processes Models of Smith, Brown-Resnick, and Schlather. The data used in this study are the rainfall intensity in the Southern Sumatra region from January 1981 to December 2021, taken from the official website of the National Aeronautics and Space Administration (NASA) Power. The results of this study show that with the RMSE criteria, the best model that can be used to predict extreme rainfall in the Southern Sumatra region for return periods of the first four years are the Smith and Brown-Resnick models, while in the fifth and sixth return periods, the Schlather model is the most accurate. Specifically, the return level results obtained with the first six return periods for each region can be seen that the Smith model is the most accurate in predicting rainfall values in Bengkulu Province, the Schlather model is the most accurate in predicting rainfall values in Jambi Province, Bangka Belitung Islands, and South Sumatra, and the Brown Resnick model is the most accurate in predicting rainfall values in Lampung Province.*

Keywords: *Extreme Rainfall, Spatial Extreme Value (SEV), Smith Model, Brown-Resnick Model, and Schlather Model*

I. INTRODUCTION

Extreme weather is a physical phenomenon of the atmosphere that is extreme and on a short-term scale [10]. The impact it causes on various life activities becomes a very serious problem. This phenomenon can occur in all parts of the world and can trigger adverse impacts on people's lives such as health problems, infrastructure paralysis, and financial losses [12]. Indonesia is a country that is often prone to experiencing extreme weather due to its climatic conditions.

The climate in Indonesia is often associated with a monsoon climate which has a considerable influence on extreme rainfall events in various regions, one of which is the Southern Sumatra region, namely Bengkulu, Lampung, South Sumatra, Jambi and the Bangka Belitung Islands. According to [1], the monsoon pattern in the southern part of Sumatra is characterized by a type of rainfall that is unimodal (one rainy season peak) where June, July and August occur during the dry season, while December, January and February are the wet months. . Meanwhile, the remaining six months is a transitional period (three months from the dry season to the rainy season and three months from the rainy season to the dry season), which of course can have a negative impact on the region itself. Therefore, to minimize the adverse impact of this event, we need a statistical method that explains the pattern of extreme rainfall events.

According to [5], one of the statistical methods developed to analyze extreme events is Extreme Value Theory (EVT). However, in multivariate data observed in several locations the EVT method cannot be used, so another method was developed, namely Spatial Extreme Value (SEV). Therefore, this research will discuss

"Extreme Rainfall Modeling Using Spatial Extreme Value with Max-Stable Processes Model Smith, Brown-Resnick, and Schlather" in the Southern Sumatra region.

II. METHODS

The climate in Indonesia is often associated with a monsoon climate which has a considerable influence on extreme rainfall events in various regions, one of which is the Southern Sumatra region, namely the Provinces of Bengkulu, Lampung, Sumatra. The data used in this study are secondary data regarding the intensity of monthly rainfall. in the Southern Sumatra region, namely the Provinces of Bengkulu, Lampung, South Sumatra, Jambi, and the Bangka Belitung Islands from January 1981 to December 2021 sourced from the official website of the National Aeronautics and Space Administration (NASA) Power. This data will be used to perform an analysis of Spatial Extreme Value with the Max-Stable Processes Model Smith, Brown-Resnick, and Schlather.

2.1 Extreme Value Theory

Extreme Value Theory (EVT) is a statistical method that can be applied to various natural phenomena, such as rainfall, floods, storms, and air pollution [11]. Two methods can be used to identify extreme values with EVT, namely the Block Maxima (BM) and Peak Over Threshold (POT) methods. The BM method is used by taking the maximum value in one period which is referred to as a block, while the POT method is used by taking the value that passes a threshold [13]. In this study identification of extreme values using the BM method.

2.2 Block Maxima Method

Block Maxima (BM) is one method used to identify extreme values based on the formation of period blocks, where observational data will be divided into certain blocks, which are based on certain time periods, for example monthly, quarterly, semester, or year. Based on the blocks formed, the maximum observed value of each block will be selected. The maximum value chosen from each block is called the extreme value [3]. An illustration of the BM method can be seen in Figure 1.

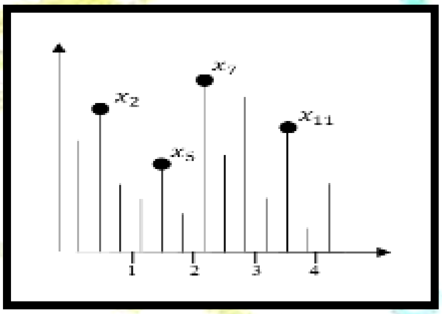

Figure 1. Ilustration of Block Maxima

According to [9], the BM method applies the Fisher-Tippet Gnedenko (1928) theorem that extreme value sample data taken from the BM method will follow the Generalized Extreme Value (GEV) distribution which has a Cumulative Distribution Function (CDF) as in the following equation:

$$F(y;\mu,\sigma,\xi) = \begin{cases} exp\left\{-\left[1+\xi\left(\frac{y-\mu}{\sigma}\right)\right]^{-\frac{1}{\xi}}\right\}, -\infty < y < \infty, \xi \neq 0, 1+\xi\left(\frac{y-\mu}{\sigma}\right) > 0 \\ exp\left\{-exp\left(-\left(\frac{y-\mu}{\sigma}\right)\right)\right\}, \quad -\infty < y < \infty, \xi = 0 \end{cases} \quad (1)$$

where y is the extreme value obtained from block maxima, μ is the location parameter with $-\infty < \mu < \infty$, σ is the scale parameter with $\sigma > 0$, and ξ is the shape parameter.

Based on ξ, the GEV distribution in Equation (1) may grouped in three type of distribution, those are Type I, for $\xi = 0$, is the Gumbel Distribution, Type II, for $\xi > 0$, is the Frechet Distribution, and Type III, for $\xi < 0$, is the Weibull Distribution [11].

2.2.1 GEV Parameter Estimation.

GEV *Probability Density Function* as follows:

$$f(y;\mu,\sigma,\xi)=\begin{cases}\frac{1}{\sigma}\left[1+\xi\left(\frac{y-\mu}{\sigma}\right)\right]^{-\frac{1}{\xi}-1}exp\left\{-\left[1+\xi\left(\frac{y-\mu}{\sigma}\right)\right]^{-\frac{1}{\xi}}\right\},\xi\neq 0\\ \frac{1}{\sigma}exp\left[-\left(\frac{y-\mu}{\sigma}\right)\right]exp\left\{-exp\left(-\left(\frac{y-\mu}{\sigma}\right)\right)\right\},\ \xi=0\end{cases}\qquad(2)$$

Maximum Likelihood Estimation (MLE) may be used to estimated μ,σ, and ξ parameter in Equation (2). But, the first derivatives of the ln *likelihood* to each parameter do not have closed form. Therefore, numerical analysis approximation are used, such as Broyden-Fletcher-Goldfarb-Shanno (BFGS). BFGS is the Quasi Newton method that improved the Newton method. The algorithm of BFGS Quasi Newton is as follows [3]:

a. Determine $\boldsymbol{\theta}^{(0)}$ the $p\times p$ null matrix, where this is also the $\boldsymbol{H}^{(k)}$ with $k=0$. p is the number of parameter.

b. Calculate the $g\left(\boldsymbol{\theta}^{(k)}\right)$ matrix and determine $S^{(k)}$.

$$g\left(\boldsymbol{\theta}^{(k)}\right)=\nabla f\left(\boldsymbol{\theta}^{(k)}\right)\qquad(3)$$

$$S^{(k)}=-\left(\boldsymbol{H}^{(k)}\right)g\left(\boldsymbol{\theta}^{(k)}\right)\qquad(4)$$

c. Determine:

$$\alpha^{(k)}=\arg\min\left[f\left(\boldsymbol{\theta}^{(k)}+\alpha^{(k)}S^{(k)}\right)\right]\qquad(5)$$

where:

$$f\left(\boldsymbol{\theta}^{(k)}+\alpha^{(k)}S^{(k)}\right)=f\left(\boldsymbol{\theta}^{(k)}\right)+\nabla f\left(\boldsymbol{\theta}^{(k)}\right)^{T}\alpha^{(k)}S^{(k)}+\frac{1}{2}\left(\alpha^{(k)}S^{(k)}\right)^{T}\boldsymbol{H}^{(k)}\left(\alpha^{(k)}S^{(k)}\right)\qquad(6)$$

d. Calculate the matrix of $\boldsymbol{H}^{(k+1)}$.

$$\boldsymbol{H}^{(k+1)}=\boldsymbol{H}^{(k)}+\left(1+\frac{\Delta g\left(\boldsymbol{\theta}^{(k)}\right)^{T}\boldsymbol{H}^{(k)}\Delta g\left(\boldsymbol{\theta}^{(k)}\right)}{\Delta g\left(\boldsymbol{\theta}^{(k)}\right)^{T}\Delta\boldsymbol{\theta}^{(k)}}\right)\frac{\Delta\boldsymbol{\theta}^{(k)}\Delta\boldsymbol{\theta}^{(k)T}}{\Delta\boldsymbol{\theta}^{(k)T}\Delta g\left(\boldsymbol{\theta}^{(k)}\right)}-\frac{\boldsymbol{H}^{(k)}\Delta g\left(\boldsymbol{\theta}^{(k)}\right)\Delta\boldsymbol{\theta}^{(k)T}+\left(\boldsymbol{H}^{(k)}\Delta g\left(\boldsymbol{\theta}^{(k)}\right)\Delta\boldsymbol{\theta}^{(k)T}\right)^{T}}{\Delta g\left(\boldsymbol{\theta}^{(k)}\right)^{T}\Delta\boldsymbol{\theta}^{(k)}}\qquad(7)$$

e. Do the numerical iteration using the following:

$$\boldsymbol{\theta}^{(k+1)}=\boldsymbol{\theta}^{(k)}+\alpha^{(k)}S^{(k)}\qquad(8)$$

f. Calculate:

$$\Delta\left(\boldsymbol{\theta}^{(k)}\right)=\boldsymbol{\theta}^{(k+1)}-\boldsymbol{\theta}^{(k)}\qquad(9)$$

g. The iteration from b until f is stopped, if $\left\|\boldsymbol{\theta}^{(k+1)}-\boldsymbol{\theta}^{(k)}\right\|\leq e$ is met or it reached the maximum number of iteration allowed.

2.2.2 Goodness-of-fit test of the GEV Distribution

The distribution goodness-of-fit test aims to identify whether the distribution (pattern) of the data corresponds to the theoretical distribution pattern. To do this, the Anderson Darling test may be used according to the following procedure [8]:

a. Hypothesis

H_0: $F(x)=F^{*}(x)$ (Data follow the theoretical distribution $F^{*}(x)$)

H_1: $F(x)\neq F^{*}(x)$ (Data do not follow the theoretical distribution $F^{*}(x)$)

b. Test statistics

$$AD=-n-\frac{1}{n}\sum_{i=1}^{n}(2i-1)\left(\ln\left(F^{*}(x_i)\right)+\ln\left(1-\left(F^{*}(x_{n+1-i})\right)\right)\right)\qquad(10)$$

where $F(x)$ is the empirical cumulatif distribution, $F^{*}(x)$ is the theoretical cumulative distribution, and n is the sample size

c. Rejection criteria

Reject H_0 if $AD>$ critical value of the statistic or $p_{value}<\alpha$.

2.3 Spatial Extreme Value

The Spatial Extreme Value (SEV) method is a method for modeling multivariate data observed at several different locations. Assuming that each component at each location is a GEV distribution, then the transformation is carried out into Frechet marginal units with the distribution function:

$$F(z) = \exp\left(-\frac{1}{z}\right), \quad z > 0 \tag{11}$$

2.4 Spatial Dependencies

The spatial dependence of extreme rainfall data can be seen by using the extremal coefficients. The extremal coefficient functions of the Smith, Brown-Resnick, and Schlather models are defined as follows [6] :

a. Smith Model

$$\theta_{Sm}(\boldsymbol{h}) = 2\Phi\left\{\sqrt{\boldsymbol{h}^T\boldsymbol{\Sigma}^{-1}\boldsymbol{h}}/2\right\} \tag{12}$$

b. Brown-Resnick

$$\theta_B(\boldsymbol{h}) = 2\Phi\{\sqrt{\gamma(\boldsymbol{h})/2}\} \tag{13}$$

c. Schlather

$$\theta_{Sc}(\boldsymbol{h}) = 1 + \sqrt{\frac{1-\rho(\boldsymbol{h})}{2}} \tag{14}$$

If the extremal coefficient value $\theta(\boldsymbol{h}) = 1$, then there are full dependencies; if the value is in the interval $1 < \theta(\boldsymbol{h}) < 2$, then it can be said that between the two regions have a dependent relationship; and if value $\theta(\boldsymbol{h}) \geq 2$, then it can be said that between the two regions have an independent relationship [7].

2.5 Max Stable Process (MSP)

Max Stable Process (MSP) is a process used to model extreme values observed spatially [2]. Modeling with MSP can use the Smith, Brown-Resnick, and Schlather models where all data will be transformed to the Frechet distribution. The process of transforming data into the Frechet distribution by using its distribution function in Equation (11), is referred to as the max-stable process. This process can be obtained by transforming Y as extreme data into the following equation:

$$Z(s) = exp\left\{-\left[1+\xi\left(\frac{y-\mu}{\sigma}\right)\right]^{-\frac{1}{\xi}}\right\} \tag{15}$$

2.5.1 Smith's Storm Profile Model

The equation of the Smith model is as follows:

$$Z(s) = \max_{i=1}^{\infty} \zeta_i \varphi(s - U_i) \tag{16}$$

The Smith model has a bivariate cumulative distribution function as follows [15]:

$$F(z_i, z_j) = exp\left\{-\frac{1}{z_i}\Phi\left(\frac{a(\boldsymbol{h})}{2} + \frac{1}{a(\boldsymbol{h})}log\left(\frac{z_j}{z_i}\right)\right) - \frac{1}{z_j}\Phi\left(\frac{a(\boldsymbol{h})}{2} + \frac{1}{a(\boldsymbol{h})}log\left(\frac{z_i}{z_j}\right)\right)\right\} \tag{17}$$

where Φ is the cumulative distribution function of the normal distribution and $a(\boldsymbol{h})$ is equal to $\sqrt{\boldsymbol{h}^T\boldsymbol{\Sigma}^{-1}\boldsymbol{h}}$, with $\boldsymbol{\Sigma}$ is the covariance matrix.

2.5.2 Brown-Resnick Model

The equation of the Brown-Resnick model is as follows:

$$Z(s) = \max_{i=1}^{\infty} \zeta_i exp(\varepsilon_i(s) - \gamma(s)) \tag{18}$$

where ε_i normal distribution with semivariogram $\gamma(h)$ and $\varepsilon(0) = 0$. The Brown-Resnick model has the same bivariate cumulative distribution function as the Smith model with $a(\boldsymbol{h}) = \sqrt{2\gamma(\boldsymbol{h})}$, so that:

$$F(z_i, z_j) = exp\left\{-\frac{1}{z_i}\Phi\left(\frac{\sqrt{2\gamma(\boldsymbol{h})}}{2} + \frac{1}{\sqrt{2\gamma(\boldsymbol{h})}}log\left(\frac{z_j}{z_i}\right)\right) - \frac{1}{z_j}\Phi\left(\frac{\sqrt{2\gamma(\boldsymbol{h})}}{2} + \frac{1}{\sqrt{2\gamma(\boldsymbol{h})}}log\left(\frac{z_i}{z_j}\right)\right)\right\} \tag{19}$$

where Φ is the cumulative distribution function of the normal distribution.

2.5.3 Schlather Model

The equation of the Schlather model is as follows [17]:

$$Z(s) = \max_{i=1}^{\infty} U_i Y_i(s) \tag{20}$$

where U_i is the Poisson process $[0, \infty)$ and $Y_i(s)$ is an independent replication of a stochastic process $Y(s)$. The Schlather model has a bivariate cumulative distribution function as follows [17]:

$$F(z_i, z_j) = exp\left\{-\left(\frac{1}{z_i} + \frac{1}{z_j}\right)\left(1 + \sqrt{1 - 2(\rho(h) + 1)\frac{z_i z_j}{(z_i + z_j)^2}}\right)\right\} \tag{21}$$

where $\rho(h)$ is the correlation function. The next step is to calculate the max-stable model parameters using the Maximum Pairwise Likelihood Estimation.

2.6 Parameter Estimation with Maximum Pairwise Likelihood Estimation

Maximum Pairwise Likelihood Estimation (MPLE) is a parameter estimation method using a pairwise density function of two variables [3]. In the context of the spatial model the GEV distribution is defined as follows:

$$GEV(\mu(s), \sigma(s), \xi(s)) \tag{22}$$

where GEV distribution parameters follow the trend surface model with the following equation:

$$\mu(s) = \beta_{0,\mu} + \beta_{1,\mu} lon(s) + \beta_{2,\mu} lat(s) \tag{23}$$

$$\sigma(s) = \beta_{0,\sigma} + \beta_{1,\sigma} lon(s) + \beta_{2,\sigma} lat(s) \tag{24}$$

$$\xi(s) = \beta_{0,\xi} \tag{25}$$

where parameter estimation processes of β_μ, β_σ, and β_ξ are carried out based on the Probability Density Function (PDF) of each Max-Stable model using MPLE. The Pairwise Logikelihood equation for m locations is defined in the following equation:

$$l_p(\boldsymbol{\beta}, \boldsymbol{z}) = \sum_{i=1}^{m-1} \sum_{j=i+1}^{m} \log f(z_i, z_j; \boldsymbol{\beta}) \tag{26}$$

with $f(z_i, z_j; \boldsymbol{\beta})$ is the probability density function of the bivariate MSP model. Thus, the Pairwise likelihood estimate for $\boldsymbol{\beta}$ is:

$$\widehat{\boldsymbol{\beta}} = \underset{\boldsymbol{\beta}}{\operatorname{argmax}}\, l_p(\boldsymbol{\beta}; \boldsymbol{z}) \tag{27}$$

In this study, there are three models that will be applied, namely the Smith, Brown-Resnick, and Schlather models. To determine the best model of the three models applied in this study, a model selection criterion is needed, namely the Takeuchi Information Criterion (TIC)..

2.7 Best Model Selection

In this study the process of parameter estimation in the MSP model uses the Maximum Pairwise Likelihood Estimation (MPLE) approach, then the criteria for selecting a suitable model is TIC which can be formulated as follows [16]:

$$TIC = -2\, l_p(\widehat{\boldsymbol{\beta}}) + 2tr\left\{H(\widehat{\boldsymbol{\beta}})^{-1} J(\widehat{\boldsymbol{\beta}})\right\} \tag{28}$$

where $H(\widehat{\boldsymbol{\beta}})$ is an information matrix and $H(\widehat{\boldsymbol{\beta}})^{-1}$ shows the variance of the estimated parameters, and J denotes the square score statistic.

2.8 Return Level

Return level is the maximum threshold that is achieved in a certain return period (T) [9]. In extreme spatial cases, especially for max-stable processes, the return level with the T-year return period is determined by pointwise quantile with the following equation [16]:

$$z_p(s) = \hat{\mu}(s) - \frac{\hat{\sigma}(s)}{\hat{\xi}(s)}\left(1 - \left[-ln\left(1 - \frac{1}{T}\right)\right]^{\hat{\xi}(s)}\right) \quad (29)$$

where T is the return period in years and $\hat{\mu}(s)$, $\hat{\xi}(s)$, $\hat{\sigma}(s)$ are location, shape, and scale parameters.

To measure the performance of the Smith, Brown-Resnick, and Schlather models based on return levels, the Root Mean Square Error (RMSE) approach can be used. The general RMSE formula is as follows [4]:

$$RMSE = \sqrt{\frac{\sum_{i=1}^{m}(y_i - \hat{y}_i)^2}{m}} \quad (30)$$

where y_i is the actual observed value of the data testing, while $\hat{y}_i$ is the actual observed value of the testing data which is the estimated or predicted value, and m is the number of locations.

2.9 Extreme Rainfall

Based on the intensity, rainfall is divided into, low rainfall if it occurs 150-200 mm/month, moderate rainfall if it occurs 200-250 mm/month, and high rainfall occurs 250-300 mm/month. Rainfall with an intensity of >50 mm/day is a parameter of heavy rain. Meanwhile, extreme rainfall has rainfall > 100 mm / day [14].

III. Results And Discussion

3.1 Description of Rainfall in Southern Sumatra

The description of rainfall in Southern Sumatra needs to be carried out as initial information to find out the characteristics or general description of the rainfall pattern to be analyzed. Based on the results of the existing analysis, the following descriptive statistical results are obtained:

Table 1. Descriptive Statistics

No.	Province	Rainfall (mm)			Skewness	Kurtosis
		Min	Mean	Max		
1.	Bengkulu	0	237,10	727,70	0,51	2,83
2.	Jambi	0	166,50	849,00	1,09	8,19
3.	Lampung	0	149,98	643,36	0,90	3,84
4.	Sumatra Selatan	0	192,10	954,30	0,91	6,55
5.	Kepulauan Bangka Belitung	0	173,70	400,80	0,13	2,37

Based on Table 1 we can find out the pattern of data distribution in each province based on skewness and kurtosis values. The skewness value in each province is positive and the kurtosis value is not in accordance with ideal conditions, where the value is less than or more than 3. The skewness value is positive and the kurtosis value is not in accordance with ideal conditions, indicating positive skew and pointed curve tail. A positive kurtosis value also indicates that the data pattern has a heavy tail, so further analysis is needed using the extreme value theory method..

3.2 Identification of Extreme Values with Block Maxima

Identification of Extreme Values with Block Maxima In this study, the Block Maxima method was used to determine extreme values from rainfall data. Determination of extreme samples in this study will be divided into 4 blocks based on a 3-month period. January to March is the first period, April to June the second period, July to September the third period, and October to December the fourth period. Each period takes one maximum value which will be used as a sample of extreme values, where the amount of data obtained with a 3-month period block is 164 observations for each region.

3.3 GEV Parameter Estimation with Maximum Likelihood Estimation

The extreme sample data that has been obtained from the Block Maxima method will then be used to estimate the univariate GEV parameters, those are μ, σ, and ξ using the MLE method. The GEV parameter estimation results are presented in the following table:

Table 2. GEV Parameter Estimation

No.	Province	$\hat{\mu}$	$\hat{\sigma}$	$\hat{\xi}$
1.	Bengkulu	266,5171	127,2138	-0,2140
2.	Jambi	193,2961	78,7941	-0,2787
3.	Kepulauan Bangka Belitung	200,9697	92,7267	-0,4197
4.	Lampung	173,3251	102,9666	-0,0969
5.	Sumatra Selatan	217,8321	111,0425	-0,2449

Table 2 shows that based on the estimated parameter values obtained, it can be concluded that the distribution of extreme rainfall data in the southern part of Sumatra follows the Weibull distribution due to the value of $\hat{\xi} < 0$. Furthermore, the estimated value of the GEV parameter that has been obtained will be used to test the suitability of the GEV distribution, with the Anderson Darling test.

3.4 Distribution Suitability Test

One method that can be used to test the fit of the GEV distribution for extreme data is to use the Anderson Darling test. Following are the results of the distribution suitability test:

Table 3. The results of the distribution fit test

No.	Province	A^2	p_{value}	Conclusion
1.	Bengkulu	0,1658	0,0870	Fail to Reject H_0
2.	Jambi	0,2480	0,3105	Fail to Reject H_0
3.	Kepulauan Bangka Belitung	0,2127	0,1671	Fail to Reject H_0
4.	Lampung	0,2411	0,3248	Fail to Reject H_0
5.	Sumatera Selatan	0,2903	0,4596	Fail to Reject H_0

Based on the results of the Anderson Darling test in Table 3 shows the conclusions Fail to Reject H_0. It means that, extreme data from each location that has been obtained by the Block Maxima method follows the theoretical distribution (GEV).

3.5 Spatial Dependencies

Identification of these spatial dependencies is the first step in spatial extreme value analysis using the max stable processes approach, where the extremal coefficient is a value that can be used to determine whether or not spatial dependencies exist. The following shows the extremal coefficient values and distances from each pair of locations:

Table 4. Extremal Coefficient

No.	Pairing Two Provinces	Distance	Extremal Coefficient		
			Smith	*Brown Resnick*	*Schlather*
1.	Bengkulu-Jambi	2,5511	1,4201	1,8348	1,3942
2.	Bengkulu-Kepulauan Bangka Belitung	4,0370	1,3263	1,8968	1,3030
3.	Bengkulu-Lampung	2,9648	1,4007	1,8565	1,3762
4.	Bengkulu-Sumatra Selatan	1,9932	1,3336	1,7970	1,3101
5.	Jambi-Kepulauan Bangka Belitung	2,4740	1,3906	1,8303	1,3652
6.	Jambi-Lampung	3,5871	1,4443	1,8821	1,4181
7.	Jambi-Sumatra Selatan	1,6938	1,3343	1,7709	1,3098
8.	Kepulauan Bangka Belitung-Lampung	2,8491	1,3284	1,8509	1,3052
9.	Kepulauan Bangka Belitung-Sumatra Selatan	2,0628	1,2798	1,8024	1,2571
10.	Lampung-Sumatra Selatan	1,8989	1,2769	1,7892	1,2545

3.6 Spatial GEV Parameter Estimation

In this study the method used to estimate Spatial GEV parameters is Maximum Pairwise Likelihood Estimation (MPLE). For each parameter estimate of $\hat{\mu}$, $\hat{\sigma}$, and $\hat{\xi}$ will be calculated using a trend surface model, with a combination of latitude and longitude spatial components. Following are the results of parameter estimation of the spatial GEV model from the 9 combinations of trend surface models formed:

Table 5. Trend surface models and Takeuchi Information Criteria

No.	Trend surface models			TIC
1.	$\hat{\mu}(s)$	=	$0{,}9927 - 0{,}0119 lon(s) + 0{,}0012 lat(s)$	3037,96
	$\hat{\sigma}(s)$	=	$0{,}9952 - 0{,}0242 lon(s) + 0{,}0014 lat(s)$	
	$\hat{\xi}(s)$	=	$1{,}018$	
2.	$\hat{\mu}(s)$	=	$0{,}9927 - 0{,}0118 lon(s)$	3037,04
	$\hat{\sigma}(s)$	=	$0{,}9952 - 0{,}0241\ lon(s) - 0{,}0001 lat(s)$	
	$\hat{\xi}(s)$	=	$1{,}018$	
3.	$\hat{\mu}(s)$	=	$0{,}9927 - 0{,}0118 lon(s) + 0{,}0002 lat(s)$	3036,94
	$\hat{\sigma}(s)$	=	$0{,}9952 - 0{,}0241\ lon(s)$	
	$\hat{\xi}(s)$	=	$1{,}018$	
4.	$\hat{\mu}(s)$	=	$0{,}9927 + 0{,}0005 lat(s)$	3037,25
	$\hat{\sigma}(s)$	=	$0{,}9951 - 0{,}0094 lon(s) + 0{,}0006 lat(s)$	
	$\hat{\xi}(s)$	=	$1{,}017$	
5.	$\hat{\mu}(s)$	=	$0{,}9930 + 0{,}0055 lon(s) + 0{,}0003 lat(s)$	3037,15
	$\hat{\sigma}(s)$	=	$0{,}9955 + 0{,}0004 lat(s)$	
	$\hat{\xi}(s)$	=	1.017	
6.	$\hat{\mu}(s)$	=	$0{,}9927 - 0{,}0118\ lon(s)$	3036,56
	$\hat{\sigma}(s)$	=	$0{,}9952 - 0{,}0241\ lon(s)$	
	$\hat{\xi}(s)$	=	$1{,}018$	
7.	$\hat{\mu}(s)$	=	$0{,}9938 + 0{,}0010\ lat(s)$	3036,78
	$\hat{\sigma}(s)$	=	$0{,}9962 + 0{,}0012\ lat(s)$	
	$\hat{\xi}(s)$	=	$1{,}015$	
8.	$\hat{\mu}(s)$	=	$0{,}9930 + 0{,}0055\ lon(s)$	3036,26
	$\hat{\sigma}(s)$	=	$0{,}9955 + 0{,}00003 lat(s)$	
	$\hat{\xi}(s)$	=	$1{,}017$	
9.	$\hat{\mu}(s)$	=	$0{,}9927 + 0{,}0001\ lat(s)$	3036,26
	$\hat{\sigma}(s)$	=	$0{,}9951 - 0{,}0094\ lon(s)$	
	$\hat{\xi}(s)$	=	$1{,}017$	

Table 5 provides information on the nine trend surface model combinations formed. Based on the nine models formed, the 8th model is the best model because it has the smallest TIC value, which is 3036.26, with the model as follows:

$$\hat{\imath}(s) = 0{,}9930 + 0{,}0055\ lon(s)$$
$$\hat{\sigma}(s) = 0{,}9955 + 0{,}00003 lat(s)$$
$$= 1{,}017$$

Based on the best model formed, it will be used to estimate the spatial extreme value model parameters with max-stable processes. The parameter estimates obtained will be in the form of $\hat{\mu}$, $\hat{\sigma}$, and $\hat{\xi}$ for each provinces.

3.6.1 Smith Model

By using the BFGS method and the best trend surface model that has been obtained, the Smith model is obtained as follows:

$$\hat{\mu}(s) = 0{,}9623 + 0{,}0109\ lon(s)$$
$$\hat{\sigma}(s) = 1{,}0154 + 0{,}0039\ \ lat(s)$$
$$\hat{\xi}(s) = 1{,}184$$

Based on existing models, the Smith Model parameter estimates for each region are obtained as in the following table,

Table 6. Smith model parameter estimation

Province	$\hat{\mu}(s)$	$\hat{\sigma}(s)$	$\hat{\xi}(s)$
Bengkulu	2,0779	1,0004	1,1839

Jambi	2,0925	1,0090	1,1839
Kepulauan Bangka Belitung	2,1186	1,0065	1,1839
Lampung	2,1079	0,9959	1,1839
Sumatra Selatan	2,0987	1,0027	1,1839

3.6.2 Brown Resnick Model

By using the BFGS method and the best trend surface model that has been obtained, the Brown Resnick model is obtained as follows:

$$\hat{\mu}(s) = 0{,}9930 + 0{,}0055\ lon(s)$$
$$\hat{\sigma}(s) = 0{,}9955 + 0{,}00003\ lat(s)$$
$$\hat{\xi}(s) = 1{,}017$$

Based on existing models, parameter estimates of the Brown-Resnick model for each region are obtained as in the following table.

Table 7. Parameter estimation of the Brown-Resnick model

Province	$\hat{\mu}(s)$	$\hat{\sigma}(s)$	$\hat{\xi}(s)$
Bengkulu	1,5577	0,9954	1,0168
Jambi	1,5651	0,9955	1,0168
Kepulauan Bangka Belitung	1,5783	0,9954	1,0168
Lampung	1,5729	0,9954	1,0168
Sumatra Selatan	1,5682	0,9954	1,0168

3.6.3 Schlather Model

In the Schlather model there are several correlation functions that can be used in the model, namely Whittle-Matern, Cauchy, Powered Exponential, and Bessel. The estimated value of the dependency parameter of each of these correlation functions can be seen in the following table:

Table 8. Schlather model dependency parameters

Correlation Function	Nugget	Range	Smooth	TIC
Whittle-Matern	0,00001	5,6143	0,2313	11649,39
Powered Exponential	0,00007	2,4990	1,0580	11648,51
Cauchy	0,00023	1,4154	0,7207	11648,12
Bessel	0,34300	0,3470	22,3900	11649,39

Based on Table 8, it is found that the correlation function has the smallest TIC value, namely Cauchy with a TIC value of 11648.12. Furthermore, this Cauchy correlation function will be used to estimate the parameters of the Schlather model, so that the Schlather model is obtained as follows:

$$\hat{\mu}(s) = 0{,}9884 + 0{,}0093\ lon(s)$$
$$\hat{\sigma}(s) = 1{,}0549 + 0{,}0037\ lat(s)$$
$$\hat{\xi}(s) = 1{,}1850$$

Based on existing models, parameter estimates of the Brown-Resnick model for each region are obtained as in the following table.

Table 9. Schlather model parameter estimation

Province	$\hat{\mu}(s)$	$\hat{\sigma}(s)$	$\hat{\xi}(s)$
Bengkulu	1,9406	1,0407	1,1850
Jambi	1,9530	1,0489	1,1850
Kepulauan Bangka Belitung	1,9753	1,0465	1,1850
Lampung	1,9661	1,0365	1,1850
Sumatra Selatan	1,9584	1,0429	1,1850

3.7 Return Level of Smith, Brown Resnick, and Schlather Models

Return level is the estimated value at a certain time period. Return level calculations use the formula in Equation (29) with return periods (T) of 1, 2, 3, 4, 5, and 6 years. For all return level calculation results are presented in the following table.

Table 10. Return level of Smith, Brown-Resnick, and Schlater Models

Province	Return Level Prediction					
	1	2	3	4	5	6
Smith						
Bengkulu	4,9267	10,3965	16,4477	22,9038	29,6758	36,7093
Jambi	4,9658	10,4827	16,5860	23,0976	29,9279	37,0220
Kepulauan Bangka Belitung	4,9849	10,4881	16,5763	23,0719	29,8854	36,9620
Lampung	4,9439	10,3893	16,4135	22,8407	29,5825	36,5846
Sumatra Selatan	4,9541	10,4364	16,5014	22,9723	29,7599	36,8095
Brown-Resnick						
Bengkulu	4,0535	8,1617	12,2997	16,4604	20,6385	24,8307
Jambi	4,0610	8,1695	12,3077	16,4687	20,6471	24,8396
Kepulauan Bangka Belitung	4,0742	8,1826	12,3208	16,4816	20,6600	24,8524
Lampung	4,0685	8,1766	12,3145	16,4750	20,6530	24,8451
Sumatra Selatan	4,0640	8,1723	12,3104	16,4711	20,6493	24,8417
Schlater						
Bengkulu	4,9067	10,6085	16,92061	23,6579	30,7271	38,0710
Jambi	4,9423	10,6888	17,05032	23,8403	30,9649	38,3663
Kepulauan Bangka Belitung	4,9579	10,6915	17,03872	23,8135	30,9221	38,3069
Lampung	4,9202	10,5988	16,88531	23,5952	30,6357	37,9499
Sumatra Selatan	4,9306	10,6443	16,96958	23,7209	30,8048	38,1641

After getting the return level prediction results, the next step is to determine the best model of the three existing models using the RMSE value. However, in order to be able to compare the return level prediction results with the rainfall testing data in mm/month, it is necessary to first transform the return level prediction data in Table 10 to initial data units with GEV distribution. For all the calculation results of the return level transformation are presented in the following tables:

Table 4.11 Result of return level transformation

Province	Return Level Prediction					
	1	2	3	4	5	6
Smith						
Bengkulu	438,39	500,80	534,48	556,81	573,21	586,01
Jambi	295,14	329,14	346,77	358,16	366,37	372,68
Kepulauan Bangka Belitung	309,32	339,51	353,91	362,72	368,81	373,34
Lampung	325,78	388,97	425,69	451,22	470,64	486,24
Sumatra Selatan	364,84	415,94	443,04	460,80	473,73	483,75
Brown-Resnick						
Bengkulu	420,37	481,66	513,53	534,53	549,95	562,02
Jambi	284,71	318,57	335,56	346,51	354,42	360,52
Kepulauan Bangka Belitung	299,38	330,47	344,90	353,75	359,91	364,54
Lampung	308,42	369,08	402,81	425,98	443,53	457,59
Sumatra Selatan	349,61	400,19	426,06	442,94	455,23	464,79
Schlater						
Bengkulu	438,02	502,35	548,44	558,91	575,35	588,15
Jambi	294,90	329,93	416,60	359,20	367,41	373,70
Kepulauan Bangka Belitung	309,07	340,17	374,41	363,50	369,57	374,06
Lampung	325,35	390,61	379,69	453,69	473,23	488,89
Sumatra Selatan	364,48	417,17	406,05	462,44	475,39	485,40

After getting the predicted return level from each existing region, it will then be compared with the actual data. To see the goodness of the prediction results, the RMSE value can be calculated for each existing model.

Table 12. Root Mean Squared Error for the first six years return levels

Province	Actual	Prediction			Error		
		Smith	Brown Resnick	Schlather	Smith	Brown Resnick	Schlather
One year return level (2016)							
Bengkulu	543,16	438,39	420,37	438,02	104,77	122,79	105,14
Jambi	305,86	295,14	284,71	294,90	10,72	21,15	10,96
Kepulauan Bangka Belitung	374,41	309,32	299,38	309,07	65,09	75,03	65,34
Lampung	332,23	325,78	308,42	325,35	6,45	23,81	6,88
Sumatra Selatan	311,13	364,84	349,61	364,48	53,71	38,48	53,35
RMSE					60,42	68,12	60,56
Two years return level (2016-2017)							
Bengkulu	543,16	500,80	481,66	502,35	42,36	61.50	40,81
Jambi	358,59	329,14	318,57	329,93	29,45	40.02	28,66
Kepulauan Bangka Belitung	374,41	339,51	330,47	340,17	34,90	43.94	34,24
Lampung	337,50	388,97	369,08	390,61	51,47	31.58	53.11
Sumatra Selatan	326,95	415,94	400,19	417,17	88,99	73.24	90.22
RMSE					53,76	52,30	54,07
Three years return level (2016-2018)							
Bengkulu	548,44	534,48	513,53	548,44	13,96	34,91	11,99
Jambi	416,60	346,77	335,56	416,60	69,83	81,04	68,84
Kepulauan Bangka Belitung	374,41	353,91	344,90	374,41	20,50	29,51	19,72
Lampung	379,69	425,69	402,81	379,69	46,00	23,12	48,22
Sumatra Selatan	406,05	443,04	426,06	406,05	36,99	20,01	38,55
RMSE					42,37	43,80	42,62
Four years return level (2016-2019)							
Bengkulu	548,44	556,81	534,53	558,91	8,37	13,91	10,47
Jambi	416,60	358,16	346,51	359,20	58,44	70,09	57,40
Kepulauan Bangka Belitung	384,96	362,72	353,75	363,50	22,24	31,21	21,46
Lampung	379,69	451,22	425,98	453,69	71,53	46,29	74,00
Sumatra Selatan	421,88	460,80	442,94	462,44	38,92	21,06	40,56
RMSE					46,07	41,63	46,88
Five years return level (2016-2020)							
Bengkulu	548,44	573,21	549,95	575,35	24,77	1,51	26,91
Jambi	849,02	366,37	354,42	367,41	482,65	494,60	481,61
Kepulauan Bangka Belitung	384,96	368,81	359,91	369,57	16,15	25,05	15,39
Lampung	432,42	470,64	443,53	473,23	38,22	11,11	40,81
Sumatra Selatan	421,88	473,73	455,23	475,39	51,85	33,35	53,51
RMSE					218,16	222,03	217,92
Six years return level (2016-2021)							

Bengkulu	548,44	586,01	562,02	588,15	37,57	13,58	39,71
Jambi	849,02	372,68	360,52	373,70	476,34	488,50	475,32
Kepulauan Bangka Belitung	384,96	373,34	364,54	374,06	11,62	20,42	10,90
Lampung	432,42	486,24	457,59	488,89	53,82	25,17	56.,47
Sumatra Selatan	954,33	483,75	464,79	485,40	470,58	489,54	468,93
		RMSE			300,93	309,68	300,24

Based on the return level results obtained with return periods of 1, 2, 3, 4, 5, and 6 years it shows that rainfall continues to increase from year to year and in each return period the largest monthly rainfall continues to occur in Bengkulu Province. With the RMSE criteria, for return periods of 1, 2, 3, and 4 years, it was found that the Smith and Brown Resnick model was more accurate in estimating extreme rainfall values than the Schlather model in the short term. Whereas for a long period of time, in the return period of 5 to 6 years it is found that the Schlather model is the most accurate to use in estimating extreme rainfall values, compared to the Smith and Brown Resnick model..

In addition to determining the best model based on the overall return level results, the return level prediction results will then be compared with the actual data for each region per each method. To see the goodness of the prediction results, the RMSE value can also be calculated based on each area analyzed. Following are the results of the RMSE value analysis for each region.

Table 13. Root Mean Squared Error for each Province

Periods	Rainfall (mm)				Error		
	Actual	Smith	Brown Resnick	Schlather	Smith	Brown Resnick	Schlather
			Bengkulu				
2016	543,16	438,39	420,37	438,02	104,77	122,79	105,14
2016-2017	543,16	500,80	481,66	502,35	42,36	61,50	40,81
2016-2018	548,44	534,48	513,53	536,45	13,96	34,91	11,99
2016-2019	548,44	556,81	534,53	558,91	8,37	13,91	10,47
2016-2020	548,44	573,21	549,95	575,35	24,77	1,51	26,91
2016-2021	548,44	586,01	562,02	588,15	37,57	13,58	39,71
		RMSE			50,10	58,39	50,46
			Jambi				
2016	305,86	295,14	284,71	294,90	10,72	21,15	10,96
2016-2017	358,59	329,14	318,57	329,93	29,45	40,02	28,66
2016-2018	416,60	346,77	335,56	347,76	69,83	81,04	68,84
2016-2019	416,60	358,16	346,51	359,20	58,44	70,09	57,40
2016-2020	849,02	366,37	354,42	367,41	482,65	494,60	481,61
2016-2021	849,02	372,68	360,52	373,70	476,34	488,50	475,32
		RMSE			279,62	287,75	278,94
			Kepulauan Bangka Belitung				
2016	374,41	309,32	299,38	309,07	65,09	75,03	65,34
2016-2017	374,41	339,51	330,47	340,17	34,90	43,94	34,24
2016-2018	374,41	353,91	344,90	354,69	20,50	29,51	19,72
2016-2019	384,96	362,72	353,75	363,50	22,24	31,21	21,46
2016-2020	384,96	368,81	359,91	369,57	16,15	25,05	15,39
2016-2021	384,96	373,34	364,54	374,06	11,62	20,42	10,90

		RMSE			33,58	41,73	33,28
			Lampung				
2016	332,23	325,78	308,42	325,35	6,45	23,81	6,88
2016-2017	337,50	388,97	369,08	390,61	51,47	31,58	53,11
2016-2018	379,69	425,69	402,81	427,91	46,00	23,12	48,22
2016-2019	379,69	451,22	425,98	453,69	71,53	46,29	74,00
2016-2020	432,42	470,64	443,53	473,23	38,22	11,11	40,81
2016-2021	432,42	486,24	457,59	488,89	53,82	25,17	56,47
		RMSE			48,79	28,86	50,86
			Sumatra Selatan				
2016	311,13	364,84	349,61	364,48	53,71	38,48	53,35
2016-2017	326,95	415,94	400,19	417,17	88,99	73,24	90,22
2016-2018	406,05	443,04	426,06	444,60	36,99	20,01	38,55
2016-2019	421,88	460,80	442,94	462,44	38,92	21,06	40,56
2016-2020	421,88	473,73	455,23	475,39	51,85	33,35	53,51
2016-2021	954,33	483,75	464,79	485,40	470,58	489,54	468,93
		RMSE			199,09	203,49	198,69

Based on the return level results obtained with return periods of 1, 2, 3, 4, 5, and 6 years for each region per each method, it can be seen that based on RMSE values, the Smith model is the most accurate in predicting rainfall values in Bengkulu Province, the Schlather model is the most accurate in predicting rainfall values in Jambi, Bangka Belitung Islands, and South Sumatra, and the Brown Resnick model is the most accurate in predicting rainfall values in Lampung Province..

IV. CONCLUSION

Based on the research results obtained, it can be concluded that the return level results obtained with return periods of 1, 2, 3, 4, 5, and 6 years show that rainfall continues to increase from year to year and in each return period the largest monthly rainfall continues happened in Bengkulu Province. With the RMSE criteria, the best model that can be used to predict extreme rainfall in the Southern Sumatra region for return periods of 1, 2, 3, and 4 years is the Smith and Brown Resnick model, while for the return period of 5 to 6 years it is found that the Schlather model more accurately used in estimating extreme rainfall values. Specifically, the return level results obtained with return periods of 1, 2, 3, 4, 5, and 6 years for each region per each method, it can be seen that based on RMSE values, the Smith model is the most accurate in predicting rainfall values in Bengkulu Province, the Schlather model is the most accurate in predicting rainfall values in Jambi, Bangka Belitung Islands, and South Sumatra, and the Brown Resnick model is the most accurate in predicting rainfall values in Lampung Province.

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