Analytical approximations of real-time systems with a single joint queue and preemptive priorities working under maximal load

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Abstract: *We consider a real-time multi-server system with homogeneous servers (such as overhearing devices, unmanned aerial vehicles, machine controllers, etc.) which can be maintained/programmed for different kinds of activities (e.g. passive or active). This system provides a service for real-time tasks arriving via several channels (such as communication channels, surveillance regions, assembly lines, etc.) and involves maintenance. We address the worst case analysis of the system working under maximum load with preemptive priorities assigned for servers of different activity type. We consider a model with ample maintenance facilities and single joint queue to all channels.*
We provide various analytical approximations of steady state probabilities for these real-time systems, discuss their quality, compare the results and choose the best one.

Keywords: *analytical approximations, preemptive priority, queue, real-time system.*

I. Introduction

Real-time systems (RTS) are imbedded in most modern technological structures, such as self-guided missiles, production control systems, aircraft and space stations, reconnaissance, radars, robotic and telecommunications systems, etc.

According to [1]: "*real time systems are those systems that produce results in a timely manner*", i.e. an action performed out of time limits (too late or too early) may be useless, and sometimes harmful – even if such an action or computation is functionally correct.

The data arriving to RTS is mostly incomplete or uncertain and the output may also be uncertain. That is why many of RTS include stochastic as well as dynamic components.

Many scientific communities were treating various RTS models, and there exists a rich literature covering this area.

We will focus on RTS with a *zero deadline for the beginning of job processing*. In these systems, jobs are executed immediately upon arrival, conditional on system availability. That part of the job which is not executed immediately is lost forever, and queueing of jobs (or their parts) in such systems is impossible.

The following works treat this kind of RTS. In ([2], [3]) it was proven that the non-mix policy of never relieving an operative server maximizes the availability of a multiserver single-channel RTS involving preventive maintenance and working in general regime with any arrival pattern under consideration and constant service and maintenance times. In [4] and [5] multi-server and multi-channel (identical servers and channels) RTS (with unrestricted and restricted number of maintenance facilities respectively), working under maximum load regime were treated as finite source queues ([6]). In [7] various performance measures for RTS with arbitrary number of channels operating under a maximum load regime were presented. In [8] and [9] multi-server and multi-channel RTS working in *general regime* were studied.

In [10] it was shown that even very large number of servers in RTS with ample maintenance facilities does not guarantee the maximum system availability, and optimal routing probabilities were computed analytically (for exponentially distributed service times) and via Cross Entropy (CE) [11],[12], [13] simulation approach (for generally distributed service times). These results were extended for RTS with limited maintenance facilities in [14].

RTS with priorities were studied in [15], [16] (preemptive) and [17] (non-preemptive) respectively.

In [16] a multi-server and multi-channel RTS with single joint queue *of servers* for each channel and preemptive priorities was studied and a set of balance linear equations for steady-state probabilities was obtained. Unfortunately, these equations do not have analytical solutions.

The work presented here provides various analytical approximations of steady state probabilities, using the modifications of techniques proposed in [18] and [19]. We compare the results and choose the best one.

The paper is organized as follows: In Section 2, the description of the model with preemptive priorities is presented. Section 3 provides balance equations for model with ample maintenance teams. Section 4 several approximation methods are described. In Section 5 some numerical results are presented. Finally, Section 6 is devoted to conclusions.

II. The model

The most important characteristics of RTS with a zero deadline for the beginning of job processing are summarized in [8]. A real-world problem was studied in [16]. Here we provide the formal description of RTS from [17].

The system consists of r identical *channels*. For proper performance each channel needs exactly one fixed server at *any* instant (maximum load), otherwise the information in this channel (at this specific moment) is lost. There are *N servers* (which are subject to breakdowns) in the system. A server, which is out of order, needs R_i time units of maintenance. After repair a fixed server may be of *u*-th type/class of *quality* with probability p_u *(u=1,…,m)*. These probabilities can be used as control parameters. Only *after* the repair is completed, the quality control procedure determines the quality type of fixed server. The fixed server of *u-th* type is operative for a period of time S_u before requiring R_i hours of repair. S_u and R_i are independent exponentially distributed random variables with parameters μ_u (*u=1,…,m*) and λ respectively. It is assumed that there are *K* identical maintenance facilities/teams in the system. Each team can repair exactly one server at a time. We consider the case of ample maintenance facilities ($K \geq N$), so that all N servers can be repaired simultaneously, if necessary. The duration times R_i of repair are i.r.v. exponentially distributed with parameter λ, which does not depend on the quality type of the server (neither before nor after the repair). After repair, the fixed server will either be on stand-by or operating inside the channel. There is a single joint queue of fixed *servers* to all *r channels*.

We assume that servers of the first kind of quality type have the highest priority, servers of the second quality type are the next priority in line, and so on. Finally, servers of the *m*-th quality type have the lowest priority. Server operating in any channel is *interrupted*, if another fixed server of higher priority type arrives from maintenance. When the operating server must be repaired, the fixed server with highest priority takes its place.

The system works under a maximum load (worst case) of nonstop data arrival, which is equivalent to the case of a unique job of infinite duration in each channel (a total of exactly *r* jobs in the whole system). Thus, the *nonstop* operation of the channel is needed.

If, during some period of time of length T, there is no fixed server to provide the proper operation of the channel, we will say that the part of the job/*information* of length T is lost forever.

III. Balance equations for steady state probabilities

In [16] the state of the system was defined as $(n_1,...,n_m)$, where n_u, $u=1,...,m$ is a number of fixed servers of u-th quality type (obviously, $\sum_{u=1}^{m} n_u \le N$ and $n_u \ge 0$), and the corresponding steady state probability was denoted $P_{(n_1,...,n_m)}$. There are $\binom{N+m}{m}$ states in total. The rate of arrival of fixed servers of u-th quality type from maintenance is denoted $\lambda_u = \lambda p_u$,. This system can be represented as a network with two nodes. We will focus on the case $r < N$, otherwise all fixed servers will be busy and preemptive priority regime, therefore, will not work.

The model with ample maintenance teams ($K \ge N$) was studied, and the following set of linear equations for steady state probabilities was obtained in [16].

$$P_{(n_1,...,n_u,...,n_m)}\left[\left(N-\sum_{i=1}^{m} n_i\right)\lambda+\sum_{u=1}^{m} \mu_u \cdot \min\left(n_u, \max\left(0, r-\delta_u \cdot \sum_{i=1}^{u-1} n_i\right)\right)\right]=$$
$$=\sum_{u=1}^{m}\left[\min(n_u,1)\cdot P_{(n_1,...,n_u-1,...,n_m)}\left(N+1-\sum_{i=1}^{m} n_i\right)\lambda_u\right]+ \qquad (1)$$
$$+\min\left(N-\sum_{i=1}^{m} n_i, 1\right)\cdot\sum_{u=1}^{m}\left[P_{(n_1,...,n_u+1,...,n_m)}\cdot \mu_u \cdot \min\left(n_u+1, \max\left(0, r-\delta_u \cdot \sum_{i=1}^{u-1} n_i\right)\right)\right]$$

$$\sum_{n_1=0}^{N}\sum_{n_2=0}^{N-n_1}\cdots\sum_{n_m=0}^{N-\sum_{i=1}^{m-1} n_i} P_{(n_1,...,n_m)} = 1,$$

where $$\delta_u = 1-\delta_{u1}; \delta_{u1} = \begin{cases} 0, u \ne 1 \\ 1, u = 1 \end{cases}; \lambda = \sum_{i=1}^{m} \lambda_i; \lambda_i = \lambda \cdot p_i .$$

IV. Approximations

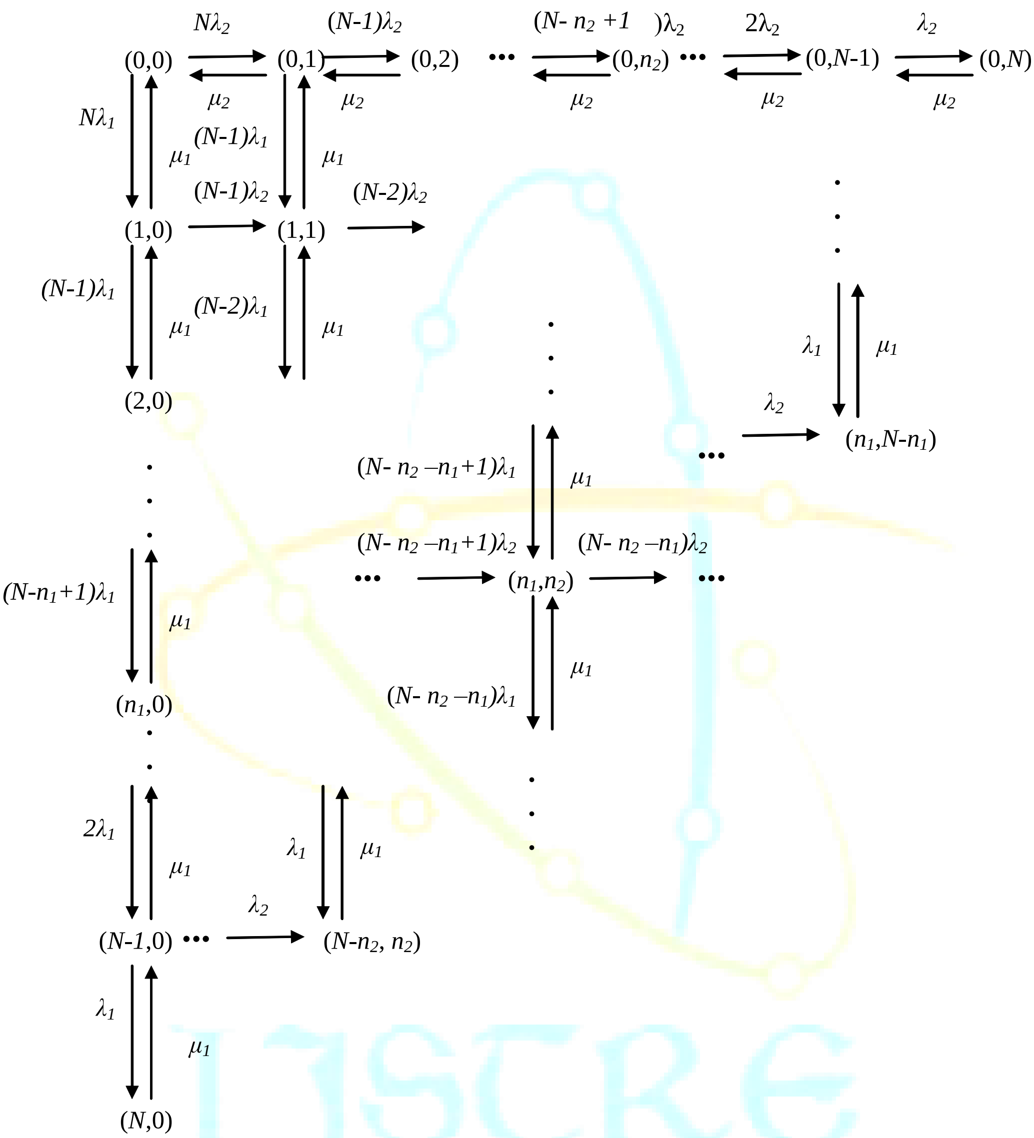
In this section we study several approximations ([18], [19]) for the model under consideration. These approximations instead of *Global Balance Equations* (1) use *Local Balance Equations* (as is shown further), and provide analytical *product form solutions* [6] for steady state probabilities. We discuss their quality and choose the best one. We will use an RSS (root of sum of squares) criteria in comparison between different approximations:

$$RSS = \sqrt{\sum_{n_1=0}^{N}\sum_{n_2=0}^{N-n_1}\cdots\sum_{n_m=0}^{N-\sum_{i=1}^{m-1} n_i} \left(P_{(n_1,...,n_m)} - \overline{P}_{(n_1,...,n_m)}\right)^2} \;, \qquad (2)$$

where $P_{(n_1,...,n_m)}$ and $\overline{P}(n_1,...,n_m)$ are exact numerical solution (of equations (1)) and analytical approximation respectively.

We will use the state transition rate diagram on Fig. 1 (see Fig 2, [16]) in order to explain approximation methods.

Fig. 1. State transition diagram of the system with $m = 2$, $r = 1 < N$, $K > N$



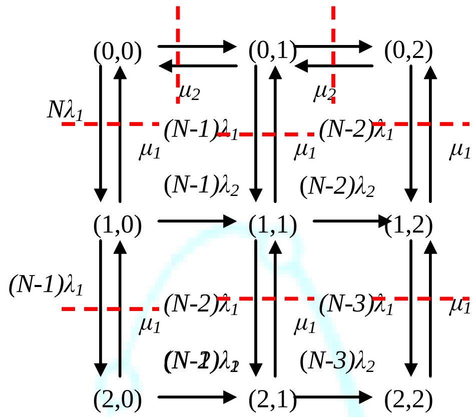
Method 1

According to [19], in order to arrive from the state (n_1, n_2) to the state $(\bar{n}_1, \bar{n}_2)$, where $(n_1 \le \bar{n}_1)$ and $(n_2 \le \bar{n}_2)$, we move first along the horizontal line n_1 to the state $(n_1, \bar{n}_2)$ using the local vertical cuts between neighbouring states for local equations.

Then we move along the vertical column $\bar{n}_2$ to the state $(\bar{n}_1, \bar{n}_2)$ using the local horizontal cuts between neighbouring states for local equations.

To be more specific, we will show the route between the states (0,0) and (2,2):
$(0,0) \longrightarrow (0,1) \longrightarrow (0,2) \longrightarrow (1,2) \longrightarrow (2,2)$ as it is shown in Figure 2, which is a part of Fig. 1.

Figure 2. The route between the states (0,0) and (2,2)



The corresponding local balance equations between neighbouring states will be as follows:

$$P_{(0,1)} = N\frac{\lambda_2}{\mu_2}P_{(0,0)}$$

$$P_{(0,2)} = (N-1)\frac{\lambda_2}{\mu_2}P_{(0,1)} = N(N-1)\left(\frac{\lambda_2}{\mu_2}\right)^2 P_{(0,0)}$$

$$P_{(1,2)} = (N-2)\frac{\lambda_1}{\mu_1}P_{(0,2)} = N(N-1)(N-2)\frac{\lambda_1}{\mu_1}\left(\frac{\lambda_2}{\mu_2}\right)^2 P_{(0,0)}$$

$$P_{(2,2)} = (N-3)\frac{\lambda_1}{\mu_1}P_{(1,2)} = N(N-1)(N-2)(N-3)\left(\frac{\lambda_1}{\mu_1}\right)^2\left(\frac{\lambda_2}{\mu_2}\right)^2 P_{(0,0)}$$

Finally, using the mathematical induction, we obtain

$$P_{(n_1,n_2)} = \left(\prod_{j=0}^{n_1+n_2-1}(N-j)\right)\left(\frac{\lambda_1}{\mu_1}\right)^{n_1}\left(\frac{\lambda_2}{\mu_2}\right)^{n_2} P_{(0,0)}. \qquad (3)$$

It can be easily seen that only two-way arcs are used in this process, while one-way arcs (like those between (1,0) and (1,1)) do not participate at all. But namely one-way arcs express the preemptive priority regime in the system. Therefore this approximation method must be adjusted for the system under consideration.

Method 2 ,
This method is a modification of Method 1. The idea is to arrive first to some of the neighbouring states of $(\bar{n}_1, \bar{n}_2)$ via two-way arcs, and only then to use one-way arcs for local balance equation for $(\bar{n}_1, \bar{n}_2)$.
In our example to reach from the state (0,0) to the state (2,2) it will work as follows. First to arrive to the states (1,2) and (2,1) according to Method 1:

$(0,0) \to (0,1) \to (0,2) \to (1,2)$

$(0,0) \to (0,1) \to (1,1) \to (2,1)$

We obtain

$$P_{(1,2)} = N(N-1)(N-2)\left(\frac{\lambda_1}{\mu_1}\right)\left(\frac{\lambda_2}{\mu_2}\right)^2 P_{(0,0)}$$

and

$$P_{(2,1)} = N(N-1)(N-2)\left(\frac{\lambda_1}{\mu_1}\right)^2\left(\frac{\lambda_2}{\mu_2}\right) P_{(0,0)}$$

respectively.

Next considering the state (2,2) and its links with the states (1,2) and (2,1) only, we obtain

$$P_{(2,2)}\mu_1 = P_{(1,2)}(N-3)\lambda_1 + P_{(2,1)}(N-3)\lambda_2.$$

Extracting $P_{(2,2)}$, and submitting $P_{(1,2)}$ and $P_{(2,1)}$ we get

$$P_{(2,2)} = N(N-1)(N-2)(N-3)\lambda_1^{\,2}\lambda_2^{\,2}\left(\frac{1}{\mu_1^{\,2}\mu_2^{\,2}} + \frac{1}{\mu_1^{\,3}\mu_2^{\,1}}\right)P_{(0,0)}.$$

Finally, using the mathematical induction, we obtain

$$P_{(n_1,n_2)} = \begin{cases} \left(\prod_{j=0}^{n_2-1}(N-j)\right)\left(\frac{\lambda_2}{\mu_2}\right)^{n_2} P_{(0,0)}, & n_1 = 0 \\ \left(\prod_{j=0}^{n_1+n_2-1}(N-j)\right)\lambda_1^{n_1}\lambda_2^{n_2}\left(\frac{1}{\mu_1^{n_1}\mu_2^{n_2}} + \frac{1}{\mu_1^{n_1+1}\mu_2^{n_2-1}}\right)P_{(0,0)}, & n_1 > 0 \end{cases} \quad .(4)$$

Method 3.

This method is a combination of the methods presented in [18] and [19].First we are treating the states on the upper line $n_1 = 0$, i.e. (0,0), (0,1), …, (0,N), while using only vertical cuts for local balance equations. Then we step down to the next horizontal line $n_1 = 1$, while using *diagonal* cuts for local balance equations (see Fig. 3). And so on we continue tovards the last horizontal line $n_1 = N$.

The corresponding local balance equations are as follows:

$$P_{(0,1)} = N\frac{\lambda_2}{\mu_2}P_{(0,0)}, P_{(0,2)} = (N-1)\frac{\lambda_2}{\mu_2}P_{(0,1)}, \ldots, P_{(0,n)} = (N-n+1)\frac{\lambda_2}{\mu_2}P_{(0,n-1)}$$

for the line $n_1 = 0$,

$$P_{(1,0)} = (N)\frac{\lambda_1}{\mu_1 + (N-1)\lambda_2}P_{(0,0)}, P_{(1,1)} = (N-1)\frac{\lambda_1}{\mu_1 + (N-2)\lambda_2}P_{(0,1)}, \ldots, P_{(1,N-1)}$$

for the line $n_1 = 1$,

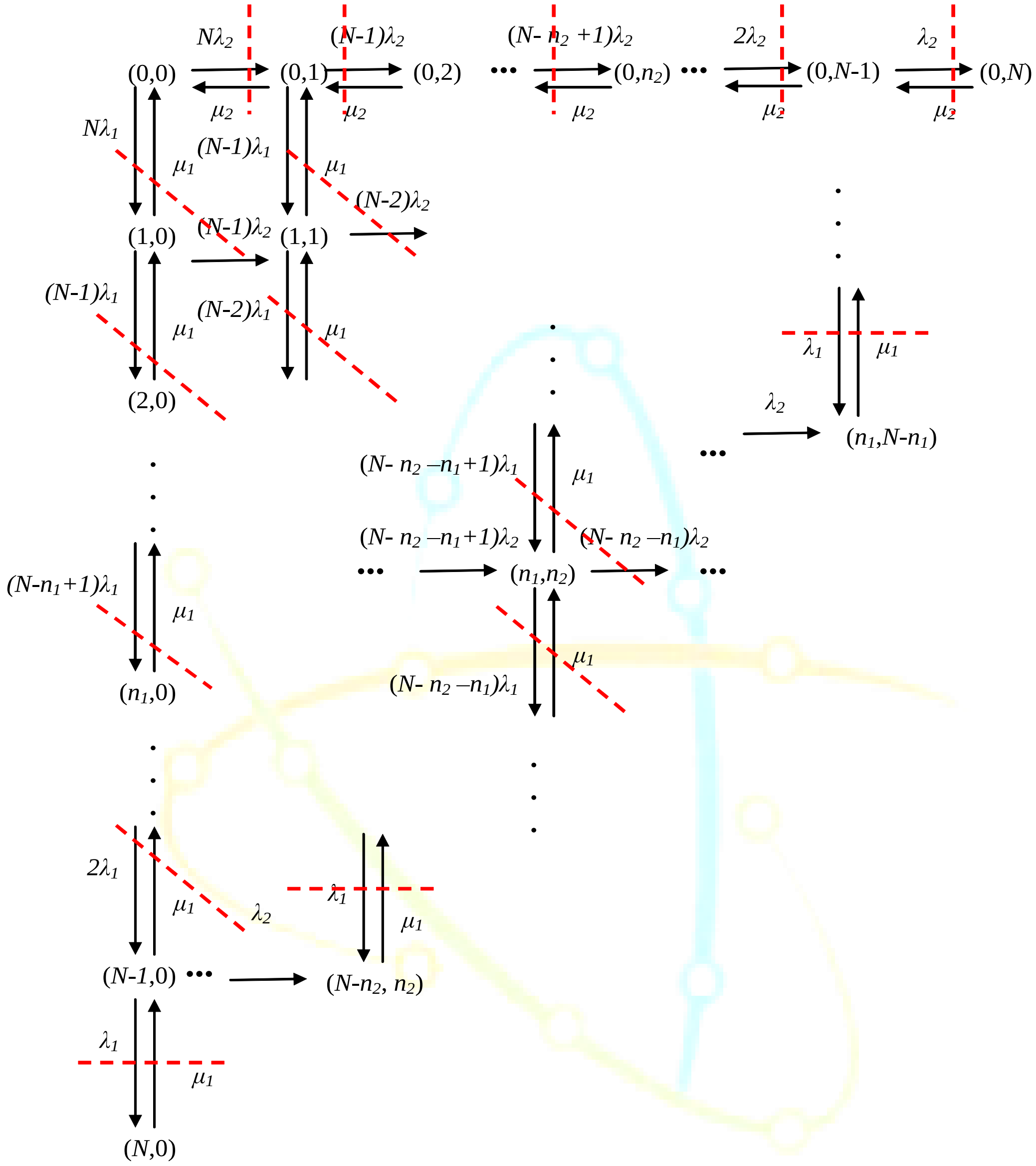
$$P_{(2,0)} = (N-1)\frac{\lambda_1}{\mu_1 + (N-2)\lambda_2}P_{(1,0)}, P_{(2,1)} = (N-2)\frac{\lambda_1}{\mu_1 + (N-3)\lambda_2}P_{(1,1)}, \ldots, P_{(2,N-2)}$$

for the line $n_1 = 2$.

Thus we arrive to the state (2,2) via the following route $(0,0) \longrightarrow (0,1) \longrightarrow (0,2) \longrightarrow (1,2) \longrightarrow (2,2)$.

Solving these equations sequentially, we get:

Figure 3. Local cuts in the system with m = 2, r = 1 < N, K > N



$$P_{(0,1)} = N\frac{\lambda_2}{\mu_2}P_{(0,0)}$$

$$P_{(0,2)} = (N-1)\frac{\lambda_2}{\mu_2}P_{(0,1)} = N(N-1)\left(\frac{\lambda_2}{\mu_2}\right)^2 P_{(0,0)}$$

$$P_{(1,2)} = (N-2)\frac{\lambda_1}{\mu_1+(N-3)\lambda_2}P_{(0,2)} = N(N-1)(N-2)\frac{\lambda_1\lambda_2{}^2}{\mu_2{}^2(\mu_1+(N-3)\lambda_2)}P_{(0,0)}$$

$$P_{(2,2)} = (N-3)\frac{\lambda_1}{\mu_1+(N-4)\lambda_2}P_{(1,2)} =$$

$$N(N-1)(N-2)(N-3)\frac{\lambda_1{}^2\lambda_2{}^2}{\mu_2{}^2(\mu_1+(N-3)\lambda_2)(\mu_1+(N-4)\lambda_2)}P_{(0,0)}$$

Finally, using the mathematical induction, we obtain

$$P_{(n_1,n_2)} = \frac{\left(\prod_{j=0}^{n_1+n_2-1}(N-j)\right)\left(\lambda_1{}^{n_1}\lambda_2{}^{n_2}\right)}{\mu_2{}^{n_2}\left(\left(\prod_{j=n_2+1}^{n_1+n_2}(\mu_1+(N-j)\lambda_2)\right)+\delta_1\right)}P_{(0,0)}, \tag{5}$$

where $\delta_1 = \begin{Bmatrix} 1, n_1 = 0 \\ 0, n_1 > 0 \end{Bmatrix}$.

Probabilities $P_{(0,0)}$ for methods 1-3 are given by the following formulae

$$P_{(0,0)} = \left[1+\sum_{n_1=0}^{N}\sum_{n_2=0}^{N-n_1}\left(\prod_{j=0}^{n_1+n_2-1}(N-j)\right)\left(\frac{\lambda_1}{\mu_1}\right)^{n_1}\left(\frac{\lambda_2}{\mu_2}\right)^{n_2}\right]^{-1}, \tag{6}$$

for Method 1,

$$P_{(0,0)} = \left[1+\sum_{n_2=0}^{N}\left(\prod_{j=0}^{n_2-1}(N-j)\right)\left(\frac{\lambda_2}{\mu_2}\right)^{n_2}+ \right.$$
$$\left. +\sum_{n_1=0}^{N}\sum_{n_2=1}^{N-n_1}\left(\prod_{j=0}^{n_1+n_2-1}(N-j)\right)\lambda_1{}^{n_1}\lambda_2{}^{n_2}\left(\frac{1}{\mu_1{}^{n_1}\mu_2{}^{n_2}}+\frac{1}{\mu_1{}^{n_1+1}\mu_2{}^{n_2-1}}\right)\right]^{-1}, \tag{7}$$

for Method 2, and

$$P_{(0,0)} = \left[1+\sum_{n_1=0}^{N}\sum_{n_2=0}^{N-n_1}\frac{\left(\prod_{j=0}^{n_1+n_2-1}(N-j)\right)\left(\lambda_1{}^{n_1}\lambda_2{}^{n_2}\right)}{\mu_2{}^{n_2}\left(\left(\prod_{j=n_2+1}^{n_1+n_2}(\mu_1+(N-j)\lambda_2)\right)+\delta_1\right)}\right]^{-1} \tag{8}$$

for Method 3 correspondingly.

V. Numerical results

In this Section, we present some numerical results, which allow us to compare between approximation methods 1-3 and to find the best one in terms of RSS.

Table 1 contains RSS in the case of ample maintenance facilities and r = 1; N = 4,5,6; λ = 10; m = 2; μ_1 = 8, μ_2 = 6; p_1 = 0.6, p_2 = 0.4.

Table 1. Comparison between three approximation methods w.r.t. RSS.

N	Method 1	Method 2	Method 3
4	0.297	0.299	**0.157**
5	0.336	0.334	**0.163**
6	0.336	0.344	**0.190**

It can be easily seen from the Table 1, that Method 3 has a minimal value of RSS, and is therefore the best one.

Tables 2-4 contain values of steady state probabilities of all three approximation methods as well as exact results in the case of ample maintenance facilities and r = 1; N = 4,5,6; λ = 10; m = 2; μ_1 = 8, μ_2 = 6; p_1 = 0.6, p_2 = 0.4.

Table 2. Steady state probabilities obtained by approximation methods and exact solution, N = 4

Exact solution	Method 3	Method 2	Method 1	States (n_1,n_2)	Exact solution	Method 3	Method 2	Method 1	States (n_1,n_2)
0.002	0.021	0.063	0.085	4,0	0.004	0.021	0.008	0.011	0,0
0.024	0.062	0.087	0.067	1,1	0.023	0.055	0.022	0.030	0,1
0.121	0.110	0.116	0.089	1,2	0.096	0.110	0.044	0.060	0,2
0.218	0.110	0.078	0.060	1,3	0.209	0.146	0.059	0.080	0,3
0.027	0.062	0.131	0.101	2,1	0.140	0.097	0.039	0.053	0,4
0.105	0.082	0.087	0.067	2,2	0.003	0.025	0.025	0.034	1,0
0.022	0.046	0.098	0.076	3,1	0.003	0.028	0.056	0.076	2,0
					0.003	0.028	0.084	0.113	3,0

Table 3. Steady state probabilities obtained by approximation methods and exact solution, $N = 5$

Exact solution	Method 3	Method 2	Method 1	States (n_1,n_2)	Exact solution	Method 3	Method 2	Method 1	States (n_1,n_2)
0.003	0.020	0.027	0.025	1,1	0.001	0.005	0.002	0.002	0,0
0.024	0.050	0.054	0.049	1,2	0.004	0.006	0.002	0.008	0,1
0.121	0.088	0.073	0.066	1,3	0.023	0.066	0.021	0.022	0,2
0.217	0.088	0.048	0.044	1,4	0.096	0.132	0.041	0.044	0,3
0.003	0.022	0.061	0.055	2,1	0.209	0.176	0.055	0.058	0,4
0.027	0.050	0.082	0.074	2,2	0.139	0.118	0.037	0.039	0,5
0.104	0.066	0.054	0.049	2,3	0.000	0.006	0.008	0.009	1,0
0.003	0.022	0.092	0.083	3,1	0.000	0.007	0.023	0.028	2,0
0.022	0.037	0.061	0.055	3,2	0.000	0.008	0.052	0.062	3,0
0.003	0.017	0.069	0.062	4,1	0.000	0.008	0.079	0.094	4,0
					0.000	0.006	0.059	0.070	5,0

Table 4. Steady state probabilities obtained by approximation methods and exact solution, $N = 6$

Exact solution	Method 3	Method 2	Method 1	States (n_1,n_2)	Exact solution	Method 3	Method 2	Method 1	States (n_1,n_2)
0.003	0.022	0.021	0.018	1,2	0.000	0.001	0.000	0.000	0,0
0.024	0.109	0.084	0.072	1,3	0.001	0.005	0.001	0.002	0,1
0.121	0.097	0.056	0.048	1,4	0.004	0.018	0.004	0.006	0,2
0.217	0.097	0.06	0.051	1,5	0.023	0.048	0.011	0.016	0,3
0.000	0.008	0.024	0.02	2,1	0.096	0.097	0.021	0.032	0,4
0.003	0.025	0.047	0.04	2,2	0.209	0.129	0.028	0.042	0,5
0.022	0.041	0.047	0.04	2,3	0.139	0.086	0.019	0.028	0,6
0.104	0.073	0.042	0.036	2,4	0.000	0.002	0.001	0.002	1,0
0.000	0.009	0.053	0.045	3,1	0.000	0.002	0.005	0.008	2,0
0.004	0.054	0.063	0.054	3,2	0.000	0.003	0.015	0.023	3,0
0.022	0.025	0.071	0.06	3,3	0.000	0.003	0.034	0.051	4,0
0.000	0.009	0.08	0.068	4,1	0.000	0.003	0.051	0.077	5,0
0.003	0.018	0.053	0.045	4,2	0.000	0.002	0.038	0.057	6,0
0.000	0.007	0.06	0.051	5,1	0.000	0.007	0.008	0.007	1,1

VI. Conclusions

We have found a good analytical approximations (Method 3, equations (5) and (8)), for RTS with a single joint queue, ample maintenance facilities and preemptive priorities working under maximal load. In our future research we will try to find good analytical approximations for RTS with a single joint queue and shortage of maintenance teams and for RTS with separate queue for each channel.

References

[1] J.A. Stankovic, (1988), Misconceptions about real-time computing: A serious problem for next generation systems, *IEEE Computer* **21**(10), 10-19.

[2] J. Kreimer and A. Mehrez, (1993), "An optimal operation policy for real-time n-server stand-by system involving preventive maintenance," *European Journal of Operational Research*, vol. **69**, 50-54,.

[3] J. Kreimer and A. Mehrez, (1994), "Optimal Real-Time Data Acquisition and Processing by a Multiserver Stand-By System," *Operations Research*, vol. **42**, no. 1, 24-30.

[4] J. Kreimer and A. Mehrez, (1998), "Computation of Availability of a Real-Time System Using Queueing Theory Methodology," *Journal of the Operational Research Society*, vol. **49**, pp.1095-1100.

[5] J. Kreimer, (1999), "Real-time multiserver and multichannel systems with shortage of maintenance crews," *Mathematical and Computer Modelling*, vol. 30, no. 11-12, pp. 169-176.

[6] D. Gross, J.F. Shortle, J.M. Thompson and C.M. Harris, (2008) *Fundamentals of Queueing Theory,* 4th edition, John Wiley, New York.

[7] J. Kreimer, "Real-time system with homogeneous servers and nonidentical channels in steady state," *Computers & Operations Research*, vol. 29, no 11, 1465-1473, 2002.

[8] J. Kreimer, "Effectiveness analysis of real-time data acquisition and processing multichannel systems," *IEEE J. on Reliability*, vol 51, no 1, 91-99, 2002.

[9] Ianovsky E. and Kreimer J**.,** "Performance of Real-Time Multiserver and Multichannel System with Joint Source", Communications in Dependability and Quality Management, Vol. 5, No 2, pp. 75-88, 2002.

[10] Ianovsky E. and Kreimer J., (2011) An Optimal Routing Policy for Unmanned Aerial Vehicles (Analytical and Cross-Entropy Simulation Approach), *Annals of Operations Research* SI MC simulation, **189**, no. 2, 215-253.

[11] Rubinstein R.Y. and Kroese D.P., (2008) Simulation and the Monte Carlo Method, 2nd edition, John Wiley & Sons, New York.

[12] Rubinstein R. Y., Ridder A. and Vaisman R., (2014) Fast Sequential Monte Carlo Methods for Counting and Optimization, John Wiley & Sons, New York.

[13] Kroese D.P., Taimre T. and Botev Z.I. (2011) Handbook of Monte Carlo Methods, John Wiley & Sons, New York.

[14] Ianovsky E. and Kreimer J., (2015), " Multiserver Multichannel Real-Time System with Limited Maintenance Facilities under Maximum Load", *Open Journal of Applied Sciences*, 2015, Nu 5, pp. 368-375. http://dx.doi.org/10.4236/ojapps.2015.57037

[15] Bassan E. and Kreimer J., (2008) Multiserver and Multichannel Real-Time Systems with Separate Queues and Preemptive priorities. *Computer Modelling and New Technologies*, **42**, No.2, 7-15.

[16] Bassan E. and Kreimer J., (2009), "Multichannel Real-Time System with Single Joint Queue and Preemptive Priorities", Communications in Dependability and Quality Management, vol. 12, No. 2, pp. 37-50.

[17] J. Kreimer**,** E. Ianovsky, (2015), "Real Time Systems with Nonpreemptive Priorities and Ample Maintenance Facilities", *Journal of Computer and Communications*, Nu 3, pp. 32-45. http://dx.doi.org/10.4236/jcc.2015.37004

[18] Kleinrock, L., (1976), "Queueing Systems Volume II: Computer Applications", New York, Wiley.

[19] Lazar A.A., (1995), Queueing Primer for Multiclass Optimal Flow Control, Telecommunication Network Control and Management (EE E6940).